tour. By this well-known mode of transformation, first employed by Lagrange, the equation (8) to be verified becomes

$$\left.\begin{aligned}&\int u\,ds\left[G\left(\frac{du}{dy}-\theta z\right)\cos(n,y)+G'\left(\frac{du}{dz}+\theta y\right)\cos(n,z)\right]\\ &\quad-\theta\iint u\,dy\,dz\left[G\frac{d^2u}{dy^2}+G'\frac{d^2u}{dz^2}\right]=0.\end{aligned}\right\}\quad(9)$$

Now the squares of the second, and of the first parenthesis, each equated to zero, give precisely and respectively the indefinite differential equation applicable to all points of a section, and the definite differential equation having reference to points on the contour; which equations I established in 1847 and in 1853, and presented* as containing, implicitly, the whole theory of the torsion of prisms having any base whatever and composed of matter whose contexture was doubly symmetrical, as in the case to which, for simplicity, we have limited ourselves in the above considerations. Moreover, in the several cases of circular, elliptic, triangular, equilateral, &c. sections, I obtained by calculation the same value for the potential of torsion from both its expressions, $\iint\Phi\,dy\,dz$ and $M\frac{\theta}{2}$.

It will be seen that the consideration of the potential or work of elasticity completely verifies the general and special results to which I was led, in a different manner, on establishing the theory of torsion. This consideration, moreover, is evidently connected with the methods of the *Mécanique Analytique* employed by Navier in 1821, and recently verified again by several mathematical physicists, amongst whom may be mentioned the regretted Clapeyron; for his theorem, after supplying the forgotten factor $\frac{1}{2}$, resolves itself to a particular case of the equation (1).

The calculation of the potential of torsion has also, in itself, a practical value; for the helical springs frequently opposed to shocks of different kinds, work almost wholly by the torsion of their threads, as I showed in 1843, and as was also remarked by Binet in 1814, and M. Giulio in 1840, and recently by railway engineers.

* *Mémoires des Savants Etrangers*, vol. xiv.; or note to the No. 156 of the *Leçons* of Navier.

XII. *Proceedings of Learned Societies.*

ROYAL SOCIETY.

[Continued from vol. xxviii. p. 479.]

June 16, 1864.—Major-General Sabine, President, in the Chair.

THE following communication was read :—

"Further Inquiries concerning the Laws and Operation of Electrical Force." By Sir W. Snow Harris, F.R.S., &c.

1. The author first endeavours to definitely express what is meant by *quantity of electricity*, *electrical charge*, and *intensity*.

By *quantity of electricity* he understands the actual amount of the unknown agency constituting electrical force, as represented by some arbitrary quantitative 'electrical' measure. By *electrical charge* he understands the quantity which can be sustained upon a given surface under a given electrometer indication. *Electrical intensity*, on the contrary, is 'the electrometer indication' answering to a given quantity upon a given surface.

2. The experiments of Le Monnier in 1746, of Cavendish in 1770, and the papers of Volta in 1779, are quoted as showing that bodies do not take up electricity in proportion to their surfaces. According to Volta, any plane surface extended in length sustains a greater charge—a result which this distinguished philosopher attributes to the circumstance that the electrical particles are further apart upon the elongated surface, and consequently further without each other's influence.

3. The author here endeavours to show that, in extending a surface in length, we expose it to a larger amount of inductive action from surrounding matter, by which, on the principles of the condenser, the intensity of the accumulation is diminished, and the charge consequently increased; so that not only are we to take into account the influence of the particles on each other, but likewise their operation upon surrounding matter.

4. No very satisfactory experiments seem to have been instituted showing the relation of quantity to surface. The quantity upon a given surface has been often vaguely estimated without any regard to a constant electrometer indication or intensity. The author thinks we can scarcely infer from the beautiful experiment of Coulomb, in consequence of this omission, that the capacity of a circular plate of twice the diameter of a given sphere is twice the capacity of the sphere, and endeavours to show, in a future part of the paper (Experiment 16), that the charge of the sphere and plate are to each other not really as 1·2, but as $1:\sqrt{2}$, that is, as the square roots of the exposed surfaces; so that we cannot accumulate twice the quantity of electricity upon the plate under the same electrometer indication.

5. On a further investigation of the laws of electrical charge, the quantity which any plane rectangular surface can receive under a given intensity is found to depend not only on the surface, but also on its linear boundary extension. Thus the linear boundary of 100 square inches of surface under a rectangle 37·5 inches long by 2·66 inches

wide, is about 80 inches; whilst the linear boundary of the same 100 square inches of surface under a plate 10 inches square is only 40 inches. Hence the charge of the rectangle is much greater than that of the square, although the surfaces are equal, or nearly so.

6. The author finds, by a rigid experimental examination of this question, that electrical charge depends upon surface and linear extension conjointly. He endeavours to show that there exists in every plane surface what may be termed an electrical boundary, having an important relation to the grouping or disposition of the electrical particles in regard to each other and to surrounding matter. This boundary, in circles or globes, is represented by their circumferences. In plane rectangular surfaces, it is their linear extension or perimeter.

If this *boundary* be constant, their electrical charge (1) varies with the square root of the surface. If the *surface* be constant, the charge varies with the square root of the boundary. If the surface and boundary both vary, the charge varies with the square root of the surface multiplied into the square root of the boundary. Thus, calling C the charge, S the surface, B the boundary, and μ some arbitrary constant depending on the electrical unit of charge, we have $C=\mu\sqrt{S.B}$, which will be found, with some exceptions, a general law of electrical charge. It follows from this formula, that if when we double the surface we also double the boundary, the charge will be also double. In this case the charge may be said to vary with the surface, since it varies with the square root of the surface, multiplied into the square root of the boundary. If therefore the surface and boundary both increase together, the charge will vary with the square of either quantity. The quantity of electricity therefore which surfaces can sustain under these conditions will be as the surface. If l and b represent respectively the length and breadth of a plane rectangular surface, then the charge of such a surface is expressed by $\mu\sqrt{2lb\,(l+b)}$, which is found to agree perfectly with experiment. We have, however, in all these cases to bear in mind the difference between *electrical charge* and *electrical intensity* (1).

7. The electrical intensity of plane rectangular surfaces is found to vary in an inverse ratio of the boundary multiplied into the surface. If the surface be constant, the intensity is inversely as the boundary. If the boundary be constant, the intensity is inversely as the surface. If both vary alike and together, the intensity is as the square of either quantity; so that if when the surface be doubled the boundary be also doubled, the intensity will be inversely as the square of the surface. The intensity of a plane rectangular surface being given, we may always deduce therefrom its electrical charge under a given greater intensity, since we only require to determine the increased quantity requisite to bring the electrometer indication up to the given required intensity. This is readily deduced, the intensity being, by a well-established law of electrical force, as the square of the quantity.

8. These laws relating to charge, surface, intensity, &c., apply more especially to continuous surfaces taken as a whole, and not to surfaces divided into separated parts. The author illustrates this by examining the result of an electrical accumulation upon a plane

rectangular surface taken as a whole, and the results of the same accumulation upon the same surface divided into two equal and similar portions distant from each other, and endeavours to show, that if as we increase the quantity we also increase the surface and boundary, the intensity does not change. If three or more separated equal spheres, for example, be charged with three or more equal quantities, and be each placed in separate connexion with the electrometer, the intensity of the whole is not greater than the intensity of one of the parts. A similar result ensues in charging any united number of equal and similar electrical jars. A battery of five equal and similar jars, for example, charged with a given quantity $=1$, has the same intensity as a battery of ten equal and similar jars charged with quantity $=2$; so that the intensity of the ten jars taken together is no greater than the intensity of one of the jars taken singly. In accumulating a double quantity upon a given surface divided into two equal and separate parts, the boundaries of each being the same, the intensity varies inversely as the square of the surface. Hence two separate equal parts can receive, taken together under the same electrometer indication, twice the quantity which either can receive alone, in which case the charge varies with the surface. Thus if a given quantity be disposed upon two equal and similar jars instead of upon one of the jars only, the intensity upon the two jars will be only one-fourth the intensity of one of them, since the intensity in this case varies with the square of the surface inversely, whilst the quantity upon the two jars under the same electrometer indication will be double the quantity upon one of them only; in which case the charge varies with the surface, the intensity being constant. If therefore as we increase the number of equal and similar jars we also increase the quantity, the intensity remains the same, and the charge will increase with the number of jars. Taking a given surface therefore in equal and divided parts, as for example four equal and similar electrical jars, the intensity is found to vary with the square of the quantity directly (the number of jars remaining the same), and with the square of the surface inversely (the number of jars being increased or diminished); hence the charge will vary as the square of the quantity divided by the square of the surface; and we have, calling C the charge, Q the quantity, and S the surface, $C=\frac{Q^2}{S^2}$; which formula fully represents the phenomenon of a constant intensity, attendant upon the charging of equal separated surfaces with quantities increasing as the surfaces; as in the case of charging an increasing number of equal electrical jars. Cases, however, may possibly arise in which the intensity varies inversely with the surface, and not inversely with the square of the surface. In such cases, of which the author gives some examples, the above formula does not apply.

9. From these inquiries it is evident, as observed by the early electricians, that conducting bodies do not take up electricity in proportion to their surfaces, except under certain relations of surface and boundary. If the breadth of a given surface be indefinitely diminished,

and the length indefinitely increased, the surface remaining constant, then, as observed by Volta, the least quantity which can be accumulated under a given electrometer indication is when the given surface is a circular plate, that is to say, when the boundary is a minimum, and the greatest when extended into a right line of small width, that is, when the boundary is a maximum. In the union of two similar surfaces by a boundary contact, as for example two circular plates, two spheres, two rectangular plates, &c., we fail to obtain twice the charge of one of them taken separately. In either case we fail to decrease the intensity (the quantity being constant) or to increase the charge (the intensity being constant), it being evident that whatever decreases the electrometer indication or intensity must increase the charge, that is to say, the quantity which can be accumulated under the given intensity. Conversely, whatever increases the electrometer indication decreases the charge, that is to say, the quantity which can be accumulated under the given intensity.

10. If the grouping or disposition of the electrical particles, in regard to surrounding matter, be such as not to materially influence external induction, then the boundary extension of the surface may be neglected. In all similar figures, for example, such as squares, circles, spheres, &c., the electrical boundary is, in relation to surrounding matter, pretty much the same in each, whatever be the extent of their respective surfaces. In calculating the charge, therefore, of such surfaces, the boundary extensions may be neglected, in which case their relative charges are found to be as the square roots of the surfaces only; thus the charges of circular plates and globes are as their diameters, the charges of square plates are as their sides. In rectangular surfaces also, having the same boundary extensions, the same result ensues, the charges are as the square roots of the surfaces. In cases of hollow cylinders and globes, in which one of the surfaces is shut out from external influences, only one-half the surface may be considered as exposed to external inductive action, and the charge will be as the square root of half the surface, that is to say, as the square root of the exposed surface. If, for example, we suppose a square plate of any given dimensions to be rolled up into an open hollow cylinder, the charge of the cylinder will be to the charge of the plate into which we may suppose it to be expanded as $1:\sqrt{2}$. In like manner, if we take a hollow globe and a circular plate of twice its diameter, the charge of the globe will be to the charge of the plate also as $1:\sqrt{2}$, which is the general relation of the charge of closed to open surfaces of the same extension. The charge of a square plate to the charge of a circular plate of the same diameter was found to be 1 : 1·13; according to Cavendish it is as 1 : 1·15, which is not far different. It is not unworthy of remark that the electrical relation of a square to a circular plate of the same diameter, as determined by Cavendish nearly a century since, is in near accordance with the formulæ $C=\sqrt{S}$ above deduced.

11. The author enumerates the following formulæ as embracing the general laws of quantity, surface, boundary extension, and intensity, practically useful in deducing the laws of statical electrical force.

Symbols.

Let C = electrical charge; Q = quantity; E = intensity, or electrometer indication; S = surface, B = boundary extension, or perimeter; Δ = direct induction; δ = reflected induction; F = force; D = distance.

Formulæ.

$C \propto S$, when S and B vary together.
$C \propto Q$, E being constant, or equal 1.
$C \propto \sqrt{S}$, B being constant, or equal 1.
$C \propto \sqrt{B}$, S being constant, or equal 1.
$C \propto \sqrt{S}.B$, when S and B vary together.
$E \propto \frac{1}{S.B}$ (Q being constant), for all plane rectangular surfaces.
$E \propto \frac{1}{B}$, S being constant, or equal 1.
$E \propto \frac{1}{S}$, B being constant, or equal 1.
$E \propto \frac{1}{S^2}$, when S and B vary together.
$C \propto \frac{1}{\sqrt{E}}$. . . . .
$E \propto Q^2$, S being constant, or equal 1.
$C \propto \frac{Q^2}{S^2}$. . . . .
In square plates, $C \propto$ with side of square.
In circular plates, $C \propto$ with diameter.
In globes, $C \propto$ with diameter.
Δ, or induction $\propto S$, all other things remaining the same.
The same for δ, or reflected induction.
In circular plates, globes, and closed and open surfaces,

$$E \propto \frac{1}{S}; \text{ or as } \frac{1}{\Delta}.$$

$$F\ (=E) \propto Q^2.$$

$$F \text{ or } E \propto \frac{1}{D^2}, \text{ S being constant.}$$

Generally we have $F \propto \frac{Q^2}{D^2}$.

12. The author calculates from these laws of charge for circles and globes a series of circular and globular measures of definite values, taking the circular inch or globular inch as unity, and calling, after Cavendish, a circular plate of an inch in diameter, charged to saturation, a circular inch of electricity; or otherwise charged to any degree short of saturation, a circular inch of electricity under a given intensity. In like manner he designates a globe of an inch in diameter a globular inch of electricity.

In the following Table are given the quantities of electricity contained in circular plates and globes, together with their respective intensities for diameters varying from ·25 to 2 inches; a circular plate of an inch diameter and $\frac{1}{5}$th of an inch thick being taken as unity, and supposed to contain 100 particles or units of charge.

Diameters, or units of charge.	Circle.		Globe.	
	Particles.	Intensity.	Particles.	Intensity.
0·25	25	0·062	35	0·124
0·50	50	0·250	70	0·500
0·75	75	0·560	105	1·120
1·00	100	1·000	140	2·000
1·25	125	1·560	175	3·120
1·40	140	1·960	196	3·920
1·50	150	2·250	210	4·500
1·60	160	2·560	224	5·120
1·75	175	3·060	245	6·120
2·00	200	4·000	280	8·000

13. The experimental investigations upon which these elementary data depend, constitute a second part of this paper. The author here enters upon a brief review of his hydrostatic electrometer, as recently perfected and improved, it being essential to a clear comprehension of the laws and other physical results arrived at.

In this instrument the attractive force between a charged and neutral disk, in connexion with the earth, is hydrostatically counterpoised by a small cylinder of wood accurately weighted, and partially immersed in a vessel of water. The neutral disk and its hydrostatic counterpoise are freely suspended over the circumference of a light wheel of 2·4 inches in diameter, delicately mounted on friction-wheels, so as to have perfectly free motion, and be susceptible of the slightest force added to either side of the balance. Due contrivances are provided for measuring the distance between the attracting disks. The balance-wheel carries a light index of straw reed, moveable over a graduated quadrantal arc, divided into 90° on each side of its centre. The neutral attracting plate of the electrometer is about $1\frac{1}{2}$ inch in diameter, and is suspended from the balance-wheel by a gold thread, over a similar disk, fixed on an insulating rod of glass, placed in connexion with any charged surface the subject of experiment. The least force between the two disks is immediately shown by the movement of the index over the graduated arc in either direction, and is eventually counterpoised by the elevation or depression in the water of the hydrostatic cylinder suspended from the opposite side of the wheel. The divisions on the graduated quadrant correspond to the addition of small weights to either side of the balance, which stand for or represent the amount of force between the attracting plates at given measured distances, with given measured quantities of electricity. This arrangement is susceptible of very great accuracy of measurement.

The experiment requires an extremely short time for its development, and no calculation is necessary for dissipation. The author carefully describes the manipulation requisite in the use of this instrument, together with its auxiliary appendages. He considers this electrometer, as an instrument of electrical research, quite invaluable, and peculiarly adapted to the measurement of electrical force.

14. Having fully described this electrometer, and the nature of its indications, certain auxiliary instruments of quantitative measure, to be employed in connexion with it, are next adverted to.

First, the construction and use of circular and globular transfer measures given in the preceding Table, by which given measured quantities of electricity may be transferred from an electrical jar (charged through a unit-jar from the conductor of an electrical machine) to any given surface in connexion with the electrometer. The electrical jar he terms a *quantity-jar*, the construction and employment of which is minutely explained, as also the construction and employment of the particular kind of unit-jar he employs.

15. Two experiments (1 and 2) are now given in illustration of this method of investigation.

Experiment 1 developes the law of attractive force as regards quantity; which is found to vary with the square of the number of circular or globular inches of electricity, transferred to a given surface in connexion with the fixed plate of the electrometer, the distance between the attracting surfaces being constant.

Experiment 2 demonstrates the law of force as regards distance between the attracting surfaces, the quantity of electricity being constant; and by which it is seen that the force is in an inverse ratio of the square of the distance between the attracting plates, the plates being susceptible of perfect inductive action. From these two experiments, taken in connexion with each other, we derive the following formula, $F \propto \frac{Q^2}{D^2}$; calling F the force, Q the quantity, and D the distance. It is necessary, however, to observe that this formula only applies to electrical attractive force between a charged and neutral body in connexion with the earth, the two surfaces being susceptible of free electrical induction, both direct and reflected.

16. The author now refers to several experiments (3, 4, 5, and 6), showing that no sensible error arises from the reflected inductive action of the suspended neutral disk of the electrometer, or from the increased surface attendant on the connexion of the surface under experiment with the fixed plate of the electrometer; as also that it is of no consequence whether the suspended disk be placed immediately over the fixed attracting plate of the electrometer, or over any point of the attracting surface in connexion with it.

17. Having duly considered these preliminary investigations, the author now proceeds to examine experimentally the laws of surface and boundary as regards plane rectangular surfaces, and to verify the formulæ $C=\sqrt{S.B}$, and $E=\frac{1}{S.B}$; in which C=charge, E=intensity, S=surface, and B=boundary.

For this purpose a series of smoothly-polished plates of copper were employed, varying from 10 inches square to 40 inches long by

2·5 to 6 inches wide, and about $\frac{1}{8}$th of an inch thick, exposing from 100 to 200 square inches of surface.

The charges (1) of these plates were carefully determined under a given electrometer indication, the attracting plates being at a constant distance.

Experiment 7. In this experiment, a copper plate 10 inches square is compared with a rectangular plate 40 inches long by 2·5 inches wide.

In these plates the surfaces are each 100 square inches, whilst the boundaries are 40 and 85 inches The boundaries may be taken, without sensible error, as 1 : 2, whilst the surfaces are the same.

On examining the charges of these plates, charge of the square plate was found to be 7 circular inches, under an intensity of 10°. Charge of the rectangular plate 10 circular inches nearly, under the same intensity of 10°. The charges therefore were as 7 : 10 nearly, that is, as 1 : 1·4 nearly, being the square roots of the boundaries, that is, as 1 : $\sqrt{2}$.

Experiment 8. A rectangular plate 37·5 inches long by 2·7 inches wide, surface 101 square inches, boundary 80·5 inches, compared with a rectangular plate 34·25 inches long by 6 inches wide, surface 205 square inches, boundary 80·5 inches.

Here the boundaries are the same, whilst the surfaces may be taken as 1 : 2.

On determining the charges of these plates, charge of the rectangular plate, surface 101 square inches was found to be 8·5 circular inches under an intensity of 8°. Charge of the plate with double surface =205 square inches, was found to be 12 circular inches under the same intensity of 8°; that is to say, whilst the surfaces are as 1 : 2, the charges are as 8·5 : 12 nearly, or as the square roots of the surfaces, that is, as 1 : $\sqrt{2}$.

Experiment 9. A rectangular plate 26·25 inches long by 4 inches wide, surface 105 square inches, boundary 60·5, compared with a rectangular plate 40 inches long by 5 inches wide, surface 200 square inches, boundary 90 inches.

Here the surfaces are as 1 : 2 nearly, whilst their boundaries are as 2 : 3.

Charge of the rectangular plate surface =105 square inches, 7 circular inches under an intensity of 10°. Charge of rectangular plate surface 200 square inches, 12 circular inches, under the same intensity of 10°. The charges therefore are as 7 : 12 nearly, or as 1 : 1·7, being as the square roots of the surfaces multiplied into the square roots of the boundaries very nearly.

Experiment 10. A square plate 10 inches square, surface 100 square inches, boundary 40 inches, compared with a rectangular plate 40 inches long by 5 inches wide, surface 200 square inches, boundary 90 inches.

Here the surfaces are double of each other, and the boundaries also double each other, or so nearly as to admit of their being considered double of each other. Charge of square plate 6 circular inches, under an intensity of 10°. Charge of rectangular plate 12 circular inches, under the same intensity of 10°. The charges, therefore, are as the square roots of the surfaces and boundaries conjointly, according to the formula $C=\sqrt{S.B}$, as also verified in the preceding experiment 9.

A double surface, therefore, having a double boundary, takes a double charge, but not otherwise. Neglecting all considerations of the boundary, therefore, the surface and boundary varying together, the charge in this case will be as the surface directly.

18. The author having verified experimentally the laws of surface and boundary, as regards plane rectangular surfaces, proceeds to consider the charges of square plates, circular plates, spheres, and closed and open surfaces generally.

Experiment 11. Plate 10 inches square, surface 100 square inches, boundary 40 inches, compared with a similar plate 14 inches square, surface 196 square inches, boundary 56 inches. Here the surfaces are as 1 : 2 nearly, whilst the boundaries are as 1 : $\sqrt{2}$ nearly.

In this case charge of square plate, surface 100 square inches, was found to be 8 circular inches under an intensity of 10°. Charge of the plate, surface 196 square inches, 11 circular inches, under the same intensity of 10°. Here the charges are as 8 : 11, whilst the surfaces may be taken as 1 : 2, that is to say (neglecting the boundary), the charges are as the square roots of the surfaces, according to the formula $C=\sqrt{S}$.

On examining the intensities of these plates, they were found to be inversely as the surfaces; thus 8 circular inches upon the plate surface 100, evinced an intensity of 10°; 8 circular inches upon the plate, surface 196, evinced an intensity of 5° only, or $\frac{1}{2}$ the former, according to the formula $E=\frac{1}{S}$.

Experiment 12. A circular plate of 9 inches diameter, surface 63·6 square inches, compared with a circular plate of 18 inches, or double that diameter, surface 254 square inches. Here the surfaces are as 1 : 4, whilst the boundaries or circumferences are as 1 : 2.

Charge of 9-inch plate, 6 circular inches, under an intensity of 10°. Charge of 18-inch plate, 12 circular inches, under the same intensity of 10°. Here the charges are as 1 : 2, whilst the surfaces are as 1 : 4; neglecting the difference of boundary, therefore, the charges, as in the preceding experiments, are as the square roots of the surfaces.

On examining the intensities of these plates, they were found to be inversely as the surfaces; thus 6 circular inches upon the 9-inch plate evinced an intensity of 10°, as just stated; 6 circular inches upon the 18-inch plate had only one-fourth the intensity, or 2°·5, being inversely as the surfaces, according to the formula $E=\frac{1}{S}$.

Experiment 13. A circular plate of 9 inches diameter, surface 63·6 square inches, compared with a circular plate of 12·72 inches diameter, surface 127·2 square inches. Here the surfaces are as 1 : 2.

Charge of 9-inch plate (surface 63·6 square inches), 5 circular inches, under an intensity of 8°. Charge of 12·72-inch plate (surface 127·2 square inches), 7 circular inches, under the same intensity of 8°. The charges here are as 5 : 7, whilst the surfaces are as 1 : 2; that is to say (neglecting the boundaries), the charges are as the square roots of the surfaces.

On examining the intensities of these plates, they were found to be, as in the preceding experiments, inversely as the surfaces.

Experiment 14. Comparison of a sphere of 4·5 inches diameter, surface 63·5 square inches, with a sphere of 9 inches, or double that diameter, surface 254 square inches.

Charge of sphere of 4·5 inches diameter (surface 63·5 square inches), 4 circular inches, under an intensity of 9°. Charge of sphere of 9 inches diameter (surface 254 square inches), 8 circular inches, under the same intensity of 9°. Here the charges are as 1 : 2, whilst the surfaces are as 1 : 4. The charges, therefore, are as the square roots of the surfaces, or as $1 : \sqrt{4}$.

On examining the intensities of these spheres, they were found to be inversely as the surfaces, or very nearly, being as 2°·5 and 9° respectively.

Experiment 15. Circular plate of 9 inches diameter compared with a sphere of the same diameter. Here the actual surfaces are 63·6 square inches for the plate, and 254 square inches for the sphere, being as 1 : 4. We have to observe, however, that one surface of the sphere is closed or shut up; consequently the exposed surfaces, electrically considered, neglecting one-half the surface of the sphere as being closed, are as 1 : 2, and the exposed surface of the plate is exactly one-half the exposed surface of the sphere.

Charge of plate 8 circular inches, under an intensity of 12°. Charge of sphere 11 circular inches, under the same intensity of 12°. The charges, therefore, are as 8 : 11, or as 1 : 1·4, the exposed surfaces being as 1 : 2. The charges, therefore, are as the square roots of the exposed surfaces.

On examining the intensities of the plate and sphere, they were found to be in an inverse ratio of the exposed surfaces, as in the former experiments.

Experiment 16. Comparison of a sphere of 7 inches diameter with a circular plate of 14 inches, or double that diameter. In this case the inner and outer surface of the sphere, taken together, are actually the same as the two surfaces of the plate. The inner surface of the sphere being closed, however, as in the last experiment, the surfaces of the sphere and plate, electrically considered, are therefore not equal, and the surface of the plate is twice the surface of the sphere. The surfaces, therefore, open to external induction are as 2 : 1.

On examining the charges of the plate and sphere, they were found to be as 10 : 14, or as 1 : 1·4, charge of sphere being 10 circular inches, under an intensity of 20°, and charge of plate being 14 circular inches, under the same intensity of 20°. The charge of the sphere, therefore, as compared with the charge of the plate, is as $1 : \sqrt{2}$, that is, as the square roots of the exposed surfaces.

On examining the intensities of the sphere and plate, they were found to be, as in the preceding experiments, in an inverse ratio of the exposed surfaces. We cannot, therefore, conclude, as already observed (4), that the capacity of the plate is twice that of the sphere.

19. The following experiments are further adduced in support of the preceding:—

Experiment 17. A copper plate 10 inches square, compared with the same plate rolled up into an open hollow cylinder, 10 inches long by 3·2 inches diameter. Here, as in the last experiments, although the surfaces are actually the same, yet, electrically considered, the plate has twice the surface of the cylinder, one surface of the cylinder being shut up.

On examining the charges of the cylinder and plate, they were found to be, as in the preceding experiments, as $1 : \sqrt{2}$; that is, as the square roots of the exposed surfaces, and the intensities in an inverse ratio of the surfaces, which seems to be a general law for closed and open surfaces.

Experiment 18. A hollow copper cube, side 5·7 inches, surface 195, compared with a hollow copper sphere of diameter equal side of cube, surface 103 square inches nearly.

On examining the charges of the sphere and cube, they were found to be as 9 : 10 nearly, charge of the sphere being 9 circular inches, under an intensity of 10°, and charge of cube being 10 circular inches, under the same intensity of 10°. The charges of a cube, and of a sphere whose diameter equals the side of the cube, approach each other, notwithstanding the differences of the surfaces, owing to the six surfaces of the cube not being in a disjointed or separated state.

20. The author observes, in conclusion, that the numerical results of the foregoing experiments, although not in every instance mathematically exact, yet upon the whole were so nearly accordant as to leave no doubt as to the law in operation. It would be in fact, he observes, assuming too much to pretend in such delicate experiments to have arrived at nearer approximations than that of a degree or two of the electrometer, or within quantities less than that of ·25 of a circular inch. If the manipulation, however, be skilfully conducted, and the electrical insulations perfect, it is astonishing how rigidly exact the numerical results generally come out.

GEOLOGICAL SOCIETY.

[Continued from vol. xxviii. p. 562.]

Nov. 23, 1864.—W. J. Hamilton, Esq., President, in the Chair.

The following communications were read:—

1. "On the occurrence of Organic Remains in the Laurentian Rocks of Canada." By Sir W. E. Logan, LL.D., F.R.S., F.G.S., Director of the Geological Survey of Canada.

The oldest known rocks of North America, composing the Laurentide Mountains in Canada, and the Adirondacks in the State of New York, have been divided by the Geological Survey of Canada