XXXIII. *On the Forms of the Lines of Electric Force and of Energy Flux in the neighbourhood of Wires leading Electric Waves.* By W. B. MORTON, *M.A., Professor of Natural Philosophy, Queen's College, Belfast* *.

§ 1. *Contents of the Paper.*

IN the case of electromagnetic waves which are guided through a dielectric by imperfectly conducting leads, it is generally stated † that the flow of energy, as defined by the Poynting vector, is nearly parallel to the conductor, but converges slowly upon it, the lines of flow striking the surface of the wire at a small angle, and part of the energy being turned into heat in the wire. Since the magnetic force is in planes perpendicular to the wire the above statement implies that the line of electric force leave the wire with a slight forward tilt; there is a large radial and a small longitudinal component.

Now an examination of the periodic vectors as worked out in detail by J. J. Thomson ‡, Sommerfeld §, and Mie ‖ brings to light the fact that there is a certain difference of phase between the radial and longitudinal components of the electric force. Therefore these components do not change sign together, and for a part of each wave-length the lines of force will be tilted *backwards*. In this region the flow of energy instead of being, as elsewhere, onward and inward, must be either onward and outward or backward and inward. The direction of the magnetic force decides between these alternatives. A flux of energy outwards across an element of surface of the wire indicates that, at this section, the energy of the magnetic field in the wire is being drawn upon, not only for the energy dissipated, but also to increase the magnetic energy in the adjoining dielectric.

It seemed of interest to examine the state of affairs in this eddy of the energy-flow. In what follows I have worked out in some detail the case of two parallel similar wires, which in some respects is simpler than that of a single wire. I have used the first approximation to Mie's complete solution for this case, which, as I have shown in former papers ¶, can be very simply deduced from the single-wire solution, and can be readily applied to more complicated cases. The main

* Communicated by the Author.
† *Cf.* Heaviside, 'Electromagnetic Theory,' vol. i. p. 79.
‡ J. J. Thomson, Recent Researches, p. 262.
§ Sommerfeld, Wied. *Ann.* lxvii. p. 233 (1899).
‖ Mie, *Ann. d. Phys.* ii. p. 202 (1900).
¶ Phil. Mag. [5] vol. l. p. 605 (1900), and [6] vol. i. p. 563 (1901).

results arrived at are as follows:—The state of affairs in the wire is governed, as is well known, by the magnitude of the quantity $a\sqrt{\frac{\mu p}{\rho}}$, where a is the radius of the wire, μ, ρ its permeability and resistivity, and $\frac{p}{2\pi}$ the frequency. When this is large we have the "skin-effect" well developed. Let us suppose for simplicity that ρ alone is altered. Then starting with the limiting case of very great conductivity and very thin skin, we find the region of eddy amounts to a quarter of each half wave-length, or say 45° in the argument of the periodic vectors. Further, as we should expect from the smallness of the dissipation of energy, the flow is outward in the eddy and onward everywhere. As the resistance of the wire is increased the extent of the eddy-region at first diminishes, reaches a minimum, and then increases again. In a case which I have worked out numerically the minimum is 18°. The anomalous flow is now partly outward and partly backward. As the resistance of the wire becomes very great the length of the eddy again approaches the limiting value of 45°, but the flow is now everywhere inward; it is backward in the eddy, forward elsewhere.

In the course of the investigation I have obtained the equations to the lines of force and of Poynting flux in the plane of the wires, inside and out. In order to show the properties of the curves I have plotted them in an exaggerated form, *i. e.* using constants of a different order of magnitude from those actually occurring in the physical cases.

§ 2. *Values of Electric and Magnetic Vectors.*

Take first the case of a *single wire* in a dielectric of permittivity K and permeability unity. Let Z, R be the longitudinal and radial components of electric force, H the magnetic force in circles round the wire. Then, measuring z along the direction of propagation and r outwards from the axis of the wire, the differential equations to be satisfied are

$$\left.\begin{aligned} -\frac{\partial H}{\partial z} &= \frac{4\pi R}{\rho}, \text{ or } K\frac{\partial R}{\partial t} \\ \frac{1}{r}\frac{\partial}{\partial r}(Hr) &= \frac{4\pi Z}{\rho}, \text{ or } K\frac{\partial Z}{\partial t} \\ \frac{\partial R}{\partial z} - \frac{\partial Z}{\partial r} &= -\mu\frac{\partial H}{\partial t}, \text{ or } -\frac{\partial H}{\partial t} \end{aligned}\right\}, \quad . \quad . \quad . \quad (1)$$

according as we are dealing with wire or dielectric.

Write the common periodic factor in the form $e^{i(mz-pt)}$, then $m=\frac{2\pi}{\lambda}+i\kappa$, where λ is the wave-length and κ the attenuation-constant.

Let
$$k_1=\frac{p}{V}=\frac{2\pi}{\lambda_0}, \quad \ldots \quad (2)$$

where λ_0 is the wave-length for the same frequency in free space, V the velocity of radiation,

$$k_2=(1+i)\sqrt{\frac{2\pi\mu p}{\rho}}, \quad \ldots \quad (3)$$

$$c^2=k_1^2-m^2=\left(\frac{2\pi}{\lambda_0}\right)^2-\left(\frac{2\pi}{\lambda}+i\kappa\right)^2. \quad \ldots \quad (4)$$

Then the equations are satisfied by the following scheme, omitting the periodic factor.

$$\begin{array}{lll} & \textit{Inside.} & \textit{Outside.} \\ Z \ldots\ldots & dJ_0(k_2r), & DK_0(cr) \\ R \ldots\ldots & -d.\frac{im}{k_2}J_1(k_2r), & -D\frac{im}{c}K_1(cr) \\ H \ldots\ldots & -d.\frac{ik_2}{\mu p}J_1(k_2r), & -D\frac{ik_1^2}{pc}K_1(cr) \end{array} \Bigg\} \quad \ldots \quad (5)$$

The J's and K's are the cylinder functions, vanishing for zero and infinite arguments respectively, d and D are constants. The argument of the J's should strictly be $\sqrt{k_2^2-m^2}.r$, but, as Thomson and Sommerfeld have shown, m is in actual cases negligible in comparison with k_2. Further, c is a very small quantity, so we can put for the K's the approximate values

$$K_0(cr)=\log\frac{2i}{\gamma cr},$$
$$K_1(cr)=\frac{1}{cr},$$

where γ is Euler's constant 1·781.

In the case of *two wires* at distance b apart, if $\frac{a^2}{b^2}$ can be neglected in comparison with unity, then, as I have shown in a former paper*, the state of affairs inside each wire is unaltered, and, outside, we get the approximate values by superposing two single wire solutions.

* Phil. Mag. l. p. 605. Wrong signs appear in the values given for m, k_2, R, and H in this paper, but the results arrived at are not affected.

Confining our attention to points in the plane of the wires we thus obtain for points between the wires the values

$$\left.\begin{array}{l} Z=D\log\frac{b-r}{r}, \\ R=-D\frac{im}{c^2}\left(\frac{1}{r}+\frac{1}{b-r}\right) \\ H=-D\frac{ik_1^2}{pc^2}\left(\frac{1}{r}+\frac{1}{b-r}\right) \end{array}\right\}, \quad \ldots \quad (6)$$

and for points not between the wires

$$\left.\begin{array}{l} Z=D\log\frac{b+r}{r}, \\ R=-D\frac{im}{c^2}\left(\frac{1}{r}-\frac{1}{b+r}\right), \\ H=-D\frac{ik_1^2}{pc^2}\left(\frac{1}{r}-\frac{1}{b+r}\right) \end{array}\right\}, \quad \ldots \quad (7)$$

The further discussion of phase-differences and lines of force is a good deal simplified by the fact that the constant $\frac{2i}{\gamma}$, which appears in "Z" for a single wire, goes out in the present case.

An application of the surface conditions leads to the equation for c^2

$$c^2\log\frac{b}{a}=\frac{k_1^2J_0(k_2a)}{k_2aJ_1(k_2a)}. \quad \ldots \quad (8)$$

§ 3. *The Relative Phases of the Components.*

We shall now investigate the phase-differences existing between the periodic magnitudes Z R H in the dielectric at the surface of a wire. As the expression for Z is real, the arguments of the complex quantities occurring in R and H will give their phase-differences in advance of Z. Let α and β represent these quantities for R H respectively, so that if at a given point of the wire we have

$$Z=Z_0\sin pt,$$

we shall have

$$R=R_0\sin(pt+\alpha),$$
$$H=H_0\sin(pt+\beta).$$

Then

$$\left.\begin{aligned}\alpha &= -\frac{\pi}{2} + \text{arg. } m - \text{arg. } (c^2) \\ \beta &= -\frac{\pi}{2} - \text{arg. } (c^2)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (9)$$

Using equation (8) and remembering that arg. $(k_2a) = \frac{\pi}{4}$, we have

$$\text{arg. } (c^2) = -\frac{\pi}{4} - \text{arg.} \frac{J_1(k_2a)}{J_0(k_2a)}. \quad . \quad . \quad . \quad (10)$$

$$\left.\begin{aligned}\alpha &= \text{arg.} \frac{J_1(k_2a)}{J_0(k_2a)} - \frac{\pi}{4} + \text{arg. } m \\ \text{and} \qquad \beta &= \text{arg.} \frac{J_1(k_2a)}{J_0(k_2a)} - \frac{\pi}{4}\end{aligned}\right\} \quad . \quad . \quad . \quad (11)$$

It is now necessary to examine the values of arg. $\frac{J_1(k_2a)}{J_0(k_2a)}$ for different values of the variable, and also arg. m.

Taking first $\frac{J_1}{J_0}$ we have for *small* values of k_2a

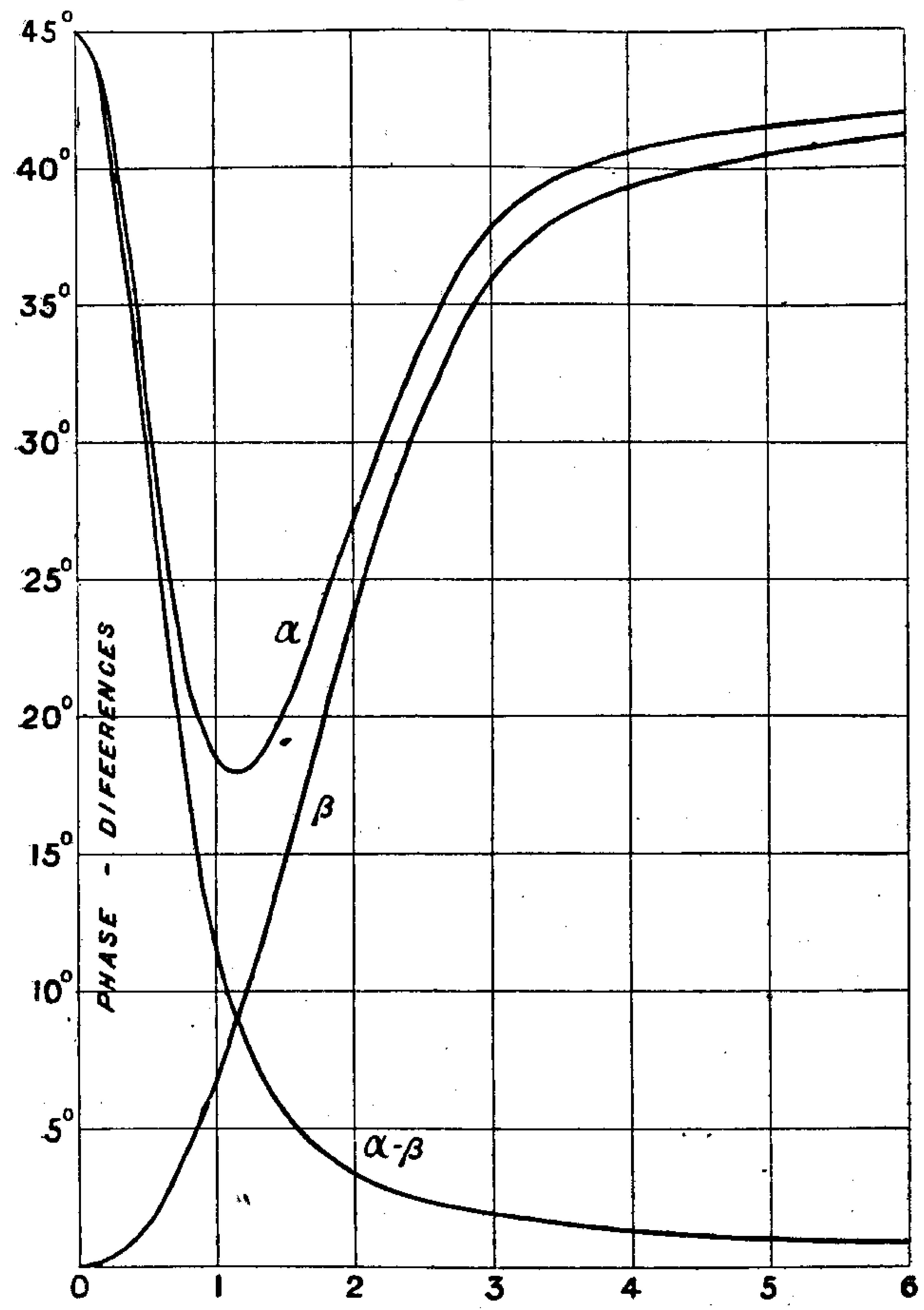
$$\frac{J_1}{J_0} = \tfrac{1}{2} k_2 a,$$

the argument is $\frac{\pi}{4}$, and $\beta = 0$.

For very *large* values $\frac{J_1}{J_0} = i$, giving $\beta = \frac{\pi}{4}$. To trace the course of the magnitude β between these limits we can use the tables of $J_0(x\sqrt{i})$ and $J_1(x\sqrt{i})$, which have been computed by Aldis *. His argument x corresponds to $2a\sqrt{\frac{\mu p \pi}{\rho}}$, and runs between values 0·1 and 6·0; the corresponding values of arg. $\frac{J_1}{J_0}$ come out 44° 23′ and 86° 10′. The curve marked β in fig. 1 is plotted by calculation from Aldis's tables.

* Aldis, Proceedings of Roy. Soc. vol. lxvi. pp. 42, 43 (1899).

Fig. 1.



$2a\sqrt{\frac{\pi\mu p}{\rho}}$.

Turning now to arg. m, equation (8) with (4) gives

$$m^2 = k_1^2 - c^2 = k_1^2 \left\{1 - \frac{J_0(k_2a)}{\log\frac{b}{a} \,.\, k_2a \,.\, J_1(k_2a)}\right\},$$

$$\therefore \text{ arg. } m = \tfrac{1}{2} \text{arg.} \left\{1 - \frac{J_0(k_2a)}{\log\frac{b}{a} \,.\, k_2a \,.\, J_1(k_2a)}\right\}. \quad . \quad (12)$$

To obtain numerical values it is necessary to assume a particular value of $\frac{b}{a}$. I have taken this as 100, and have calculated arg. m by aid of Aldis's tables. The result is shown in the curve marked $(\alpha-\beta)$ on fig. 1. The addition of the two curves obtained gives the α-curve, showing the phase-difference between R and Z.

To form an idea of the position on this diagram of actual experimental cases, we may take those given in Sommerfeld's paper as extreme cases in opposite directions. His first case, typical of skin-conduction and very small attenuation, is that of copper wire of 4 mm. diameter with frequency of 10^9 alternations per sec. This gives 1358 as the value of $2a\sqrt{\frac{\mu\pi p}{\rho}}$, and so lies far out to the right, where α and β are both practically 45°.

His other extreme case is that of platinum wire of 0·004 mm. diameter and a frequency of 3×10^8. This gives 0·256 for the determining constant; α comes on the inner side of the minimum position for α.

The quantity $(\alpha-\beta)=\text{arg. } m=\tan^{-1}\frac{k\lambda}{2\pi}$ is, for small values at least, proportional to the attenuation with given wave-length. It runs down to zero with increasing development of surface-conduction, towards the right in the diagram.

Looking now at the state of things just *inside* the wire we have of course the same phase-difference as before between H and Z. For R the advance of phase on Z is from equations (5)

$$=-\frac{\pi}{2}+\text{arg. } m-\text{arg. } k_2+\text{arg. }\frac{J_1}{J_0},$$

$$=-\frac{3\pi}{4}+\text{arg. } m-\text{arg. }\frac{J_1}{J_0},$$

$$=\alpha-\frac{\pi}{2}.$$

or the radial electric force inside is 90° behind that outside.

§ 4. *Directions of the Lines of Force and of Energy-flow at the Surface of the Wires.*

It is easy to see that when Z and R have opposite signs the lines of force are tilted backwards; when H Z are opposite the energy-flow is outward; and when H R are opposite the flow is backward. Taking, then, a half wave-length

(from 0 to π) between points at which the lengthwise electric force Z vanishes, we find the flow of energy

Outside.

$$\left.\begin{array}{ll}\text{from } 0 \text{ to } (\pi-\alpha) & \text{forward, in}\\ \text{from } (\pi-\alpha) \text{ to } (\pi-\beta) & \text{backward, in}\\ \text{from } (\pi-\beta) \text{ to } \pi & \text{forward, out}\end{array}\right\} \quad . \quad . \qquad (13)$$

Inside.

$$\left.\begin{array}{ll}\text{from } 0 \text{ to } \left(\frac{\pi}{2}-\alpha\right) & \text{backward, in}\\ \text{from } \left(\frac{\pi}{2}-\alpha\right) \text{ to } (\pi-\beta) & \text{forward, in}\\ \text{from } (\pi-\beta) \text{ to } \pi & \text{backward, out}\end{array}\right\} \quad . \qquad (14)$$

The extent of the eddy in the external energy-flow is given by α. Its course as shown by the curve is in accordance with the statement in § 1. For the limiting case of mere surface-conduction (k_2a large) $\alpha=\beta=45°$, the section $(\pi-\alpha)$ to $(\pi-\beta)$ shrinks to nothing, *i. e.* the energy-flow is everywhere in the direction of propagation of the waves. At the other extreme (k_2a small) when α again approaches 45°, β becomes zero, and the region of outward flow disappears.

§ 5. *Forms of External Lines of Force and Flow.*

We may write for points between the wires

$$\left.\begin{array}{l}Z=\log\frac{b-r}{r}e^{-kz}\sin\frac{2\pi z}{\lambda}\\ R=\frac{A}{r(b-r)}e^{-kz}\sin\left(\frac{2\pi z}{\lambda}+\alpha\right)\end{array}\right\} \quad . \quad . \quad . \qquad (15)$$

where $$A=\left|\frac{mb}{c^2}\right|.$$

The differential equation to the lines of electric force is therefore

$$\frac{dr}{dz}=\frac{A}{r(b-r)\log\frac{b-r}{r}}\cdot\frac{\sin\left(\frac{2\pi}{\lambda}z+\alpha\right)}{\sin\frac{2\pi z}{\lambda}}, \quad . \quad . \qquad (16)$$

and the integral of this

$$\frac{A\lambda}{2\pi}\left\{\cos\alpha\,.\,\frac{2\pi z}{\lambda}+\sin\alpha\log\sin\frac{2\pi z}{\lambda}\right\}$$

$$=C-\tfrac{1}{6}\{(b+2r)(b-r)^2\log(b-r)+(3b-2r)r^2\log r+br(b-r)\}. \quad (17)$$

The differential equation to the lines of the Poynting flux

$$-\frac{dz}{dr}=\frac{R}{Z}$$

gives as integral

$$\frac{b\lambda}{2\pi A}\left\{\cos\alpha\frac{2\pi z}{\lambda}-\sin\alpha\log\sin\left(\frac{2\pi z}{\lambda}+\alpha\right)\right\}$$

$$=C'+\log.\log.\frac{b-r}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

C and C′ are arbitrary constants.

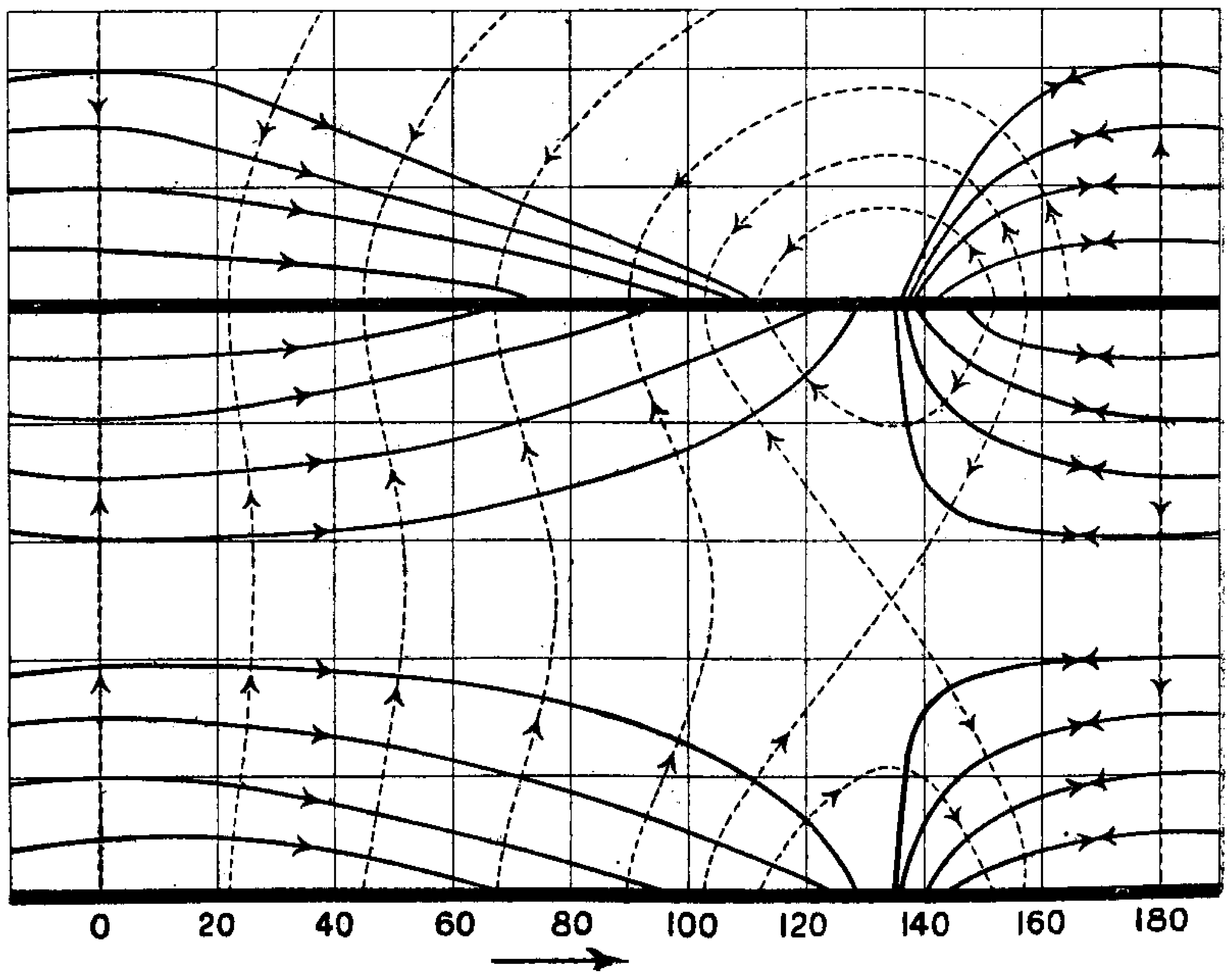
For points not between the wires the corresponding equations differ from these in having $(b-2r)$, $(3b+2r)$, and $(b+r)$ instead of the terms $(b+2r)$, $(3b-2r)$, and $(b-r)$.

In the actual case the constant A has a very large value, say order 10^4, in accordance with the fact that the lines of electric force are very nearly perpendicular to the wires *. In order to make visible the general form of the curves I have plotted fig. 2 with the following values :—

$$\alpha=45^\circ, \qquad a=1, \qquad b=100, \qquad \lambda=360,$$

$$\frac{b\lambda}{2\pi A}\cdot\frac{1}{\sqrt{2}}=\frac{1}{2}.$$

Fig. 2.



* Sommerfeld, *loc. cit.* p. 282.

The diagram thus gives a distorted picture of the arrangement of the lines in the two opposite extreme cases referred to above. The broken lines represent the lines of force and the full lines those of Poynting flux. In the eddy region from 135° to 180° the arrows on the energy lines are taken in or out according as we are dealing with a case of "k_2a" small or large, as already explained.

To pass to the actual arrangement in experimental conditions one has to imagine the sag of the lines of force reduced and the region of re-entrant lines to shrink to a very small length at 135°. So that except in the immediate neighbourhood of this point, where the radial force vanishes, the lines go practically straight across.

§ 6. *Forms of Internal Lines of Force and Flow.*

To the degree of approximation adopted in this paper, these are the same as for the case of a single wire, the return current being carried by the dielectric (Sommerfeld's case). When there is axial symmetry in the field we can, as Hertz showed *, express the quantities Z and R in the forms

$$Z=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\pi}{\partial r}\right),$$

$$R=-\frac{1}{r}\frac{\partial}{\partial z}\left(r\frac{\partial\pi}{\partial r}\right),$$

so that the lines of force are given by

$$r\frac{\partial\pi}{\partial r}=\text{const.}$$

Using this method Sommerfeld (*loc. cit.* p. 285) has given a diagram showing the course of the internal and external lines of force, with distortion, for the case of surface conduction (k_2a large). He does not draw attention to the backward tilted lines and, to judge from his figure, seems not to have plotted lines in this region.

As I wish to get the lines of energy-flow also, I shall proceed directly from the differential equation, taking separately the cases of (*a*) k_2a large, (*b*) k_2a small.

(*a*) In this case the conduction is confined to a small thickness at the surface of the wire, so that we may use for $J_0(k_2r)$ and $J_1(k_2r)$ occurring in Z, R, the forms appropriate to large values of the argument.

* Hertz, 'Electric Waves' (English translation), p. 140.

Write

$$k_2=(1+i)h, \qquad \text{so that} \quad h=\sqrt{\frac{2\pi\mu p}{\rho}}, \quad . \quad . \quad . \quad (19)$$

and let y denote the distance of a point below the surface of the wire or $(a-r)$. After a little reduction we fined for the components of electric force

$$\left.\begin{aligned} Z&=\frac{e^{-hy}}{\sqrt{a-y}}\sin\left(\frac{2\pi z}{\lambda}+hy\right), \\ R&=\frac{\pi\sqrt{2}}{h\lambda}\cdot\frac{e^{-hy}}{\sqrt{a-y}}\sin\left(\frac{2\pi z}{\lambda}+hy-\frac{\pi}{4}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad (20)$$

The attenuation constant κ is here neglected in comparison with $\frac{2\pi}{\lambda}$.

Therefore for lines of force

$$\frac{dy}{dz}=-\frac{\pi\sqrt{2}}{h\lambda}\,\frac{\sin\left(\frac{2\pi z}{\lambda}+hy-\frac{\pi}{4}\right)}{\sin\left(\frac{2\pi z}{\lambda}+hy\right)}, \quad . \quad . \quad . \quad . \quad (21)$$

leading to

$$hy=\log\sin\left(\frac{2\pi z}{\lambda}+hy+\frac{\pi}{4}\right)+\text{const.} \quad . \quad . \quad . \quad (22)$$

and for lines of flow we get

$$\begin{aligned} my(h^2+m^2)-hz(2h^2+m^2)&=hm\log\{(2h^2+m^2)\sin(mz+hy) \\ &\quad -m^2\cos(mz+hy)\}+\text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23) \end{aligned}$$

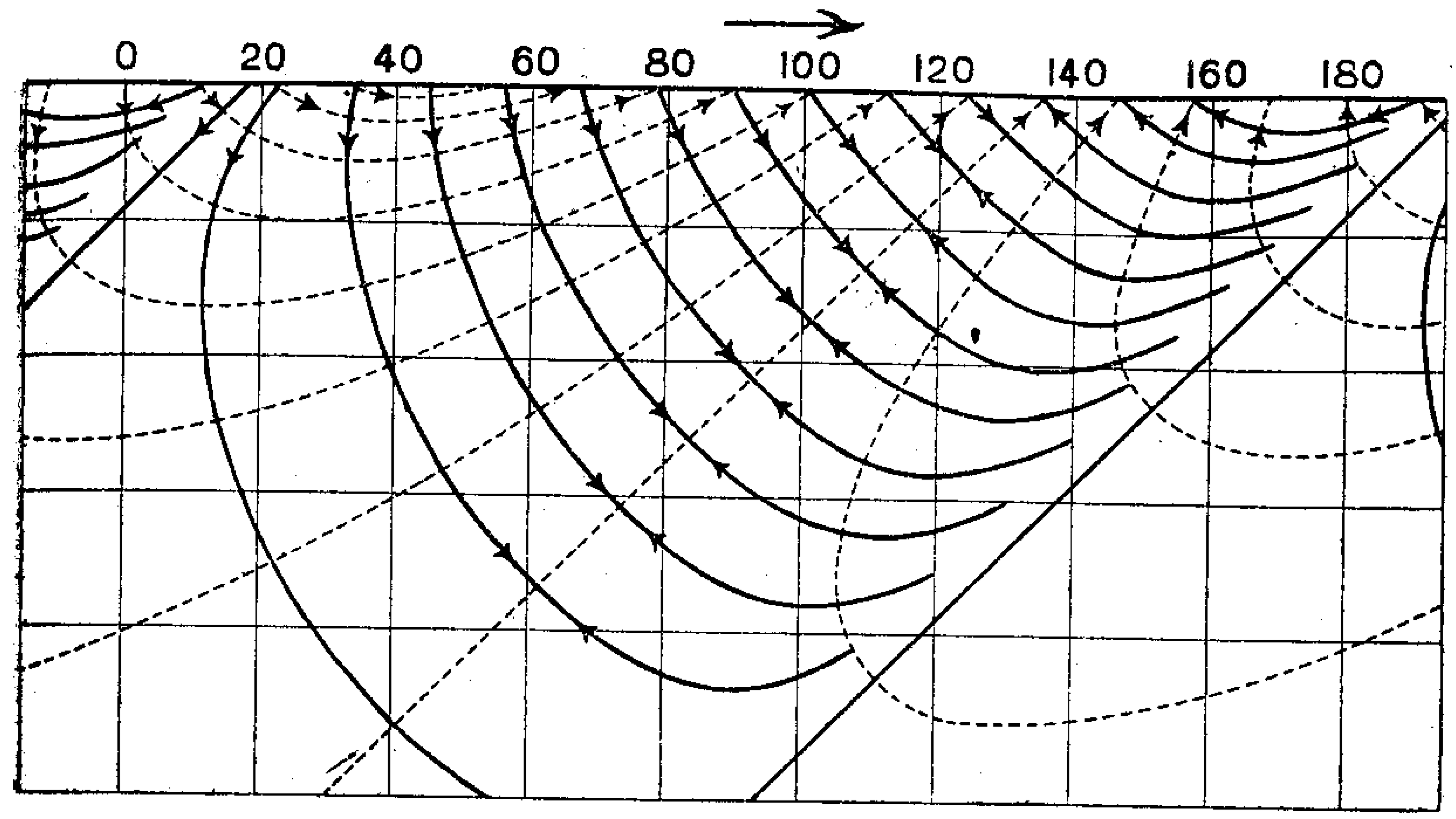
In the last equation m is written for $\frac{2\pi}{\lambda}$.

In fig. 3 these two families of curves are plotted for the simple case $m=h=1$. It will be seen that all the curves of either family can be got by moving one of them in a direction making an angle of 45° with the negative direction of z.

The angles marked on the diagram give the phase-angles of the lengthwise electric force. The magnetic force being 45° in advance of the lengthwise electric vanishes at all points of the straight line of electric force which meets the surface at 135°. The flow of energy is therefore oppositely directed on the two sides of this line, as shown by the arrows on the lines of flow. An inspection of the figure shows that the flow of energy at the surface is backward and inward from 0° to 45°, forward and inward from 45° to 135°, and backward and outward from 135° to 180°, agreeing with

(14) when $\alpha=\beta=45°$. In the actual case the lines of force are almost parallel to the surface. This is seen from (21)

Fig. 3.



when we remember that h is by hypothesis very large. Accordingly we have to imagine the straight lines of force in the diagram twisted round so as to meet the boundary at a very small angle instead of at 45°.

(*b*) *Case of k_2a very small*, the current diffused through the whole wire. Here we may put unity for $J_0(k_2r)$ and $\frac{1}{2}k_2r$ for $J_1(k_2r)$.

Further, as we have seen in § 3, the argument of m approaches 45°, so we may write

$$m=\frac{2\pi}{\lambda}(1+i).$$

So we have

$$\left.\begin{aligned} Z&=e^{\frac{-2\pi z}{\lambda}}\cdot\sin\frac{2\pi z}{\lambda} \\ R&=e^{\frac{-2\pi z}{\lambda}}\cdot\frac{\pi\sqrt{2}}{\lambda}\cdot r\cdot\sin\left(\frac{2\pi z}{\lambda}-\frac{\pi}{4}\right) \end{aligned}\right\} \quad . \quad . \quad (24)$$

It is remarkable that h disappears from the expressions, or in other words, as we approach this limit the properties of the wire cease to have an effect on the distribution of the field inside.

The equation to the lines of force is

$$\frac{dr}{dz}=r\cdot\frac{\pi\sqrt{z}}{\lambda}\cdot\frac{\sin\left(\frac{2\pi z}{\lambda}-\frac{\pi}{4}\right)}{\sin\frac{2\pi z}{\lambda}}, \quad . \quad . \quad . \quad . \quad . \quad (25)$$

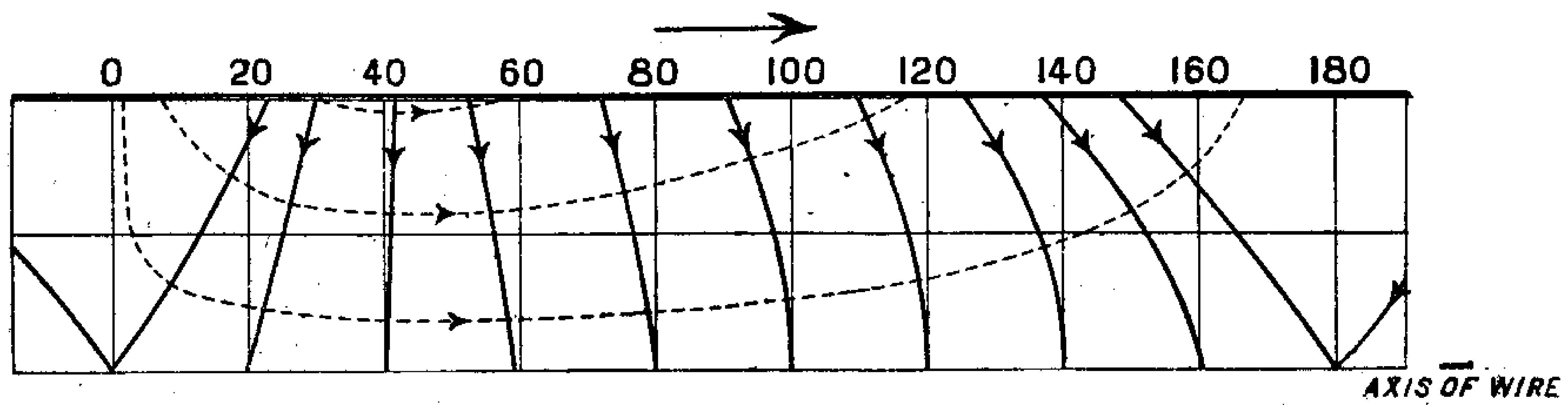
$$2\log r=\frac{2\pi z}{\lambda}-\log\sin\frac{2\pi z}{\lambda}+\text{const.} \quad . \quad . \quad . \quad (26)$$

The equation to the lines of flow comes out

$$-\frac{2\pi^2}{\lambda^2}r^2=\frac{2\pi z}{\lambda}+\log\sin\left(\frac{2\pi z}{\lambda}-\frac{\pi}{4}\right). \quad . \quad . \quad (27)$$

These curves are shown on fig. 4, beginning, in this

Fig. 4.



case, from the *axis* of the wire. It may be noted that if we plot one line of force the others are got simply by extending proportionally the ordinates measured from the axis.

In this case the magnetic force vanishes along with the longitudinal electric. The energy-flow is backward and inward from 0° to 45°, and forward and inward for the remainder of the half wave-length. The exaggeration of the diagram in this case consists in making the radius of the wire much too great in comparison with the wave-length, in order to get room to show the trend of the curves.

Queen's College, Belfast,
20th June, 1902.

XXXIV. *Deviable Rays of Radioactive Substances.* By E. Rutherford, *M.A.*, *D.Sc.*, *Macdonald Professor of Physics, and* A. G. Grier, *M.Sc.*, *Demonstrator in Physics, McGill University, Montreal* *.

§ 1. THE experiments † of Giesel, Becquerel, Curie, Meyer, and Schweidler have shown that radium gives out some rays deflectable by a magnet.

Becquerel, in addition, has shown that uranium, and the excited radioactivity due to radium, also emit rays deviable by a magnetic field. Becquerel has employed the photographic method for detecting deviable rays, while the Curies, Meyer, and Schweidler have used the electrical method for analysis of the deviable rays from radium.

Further experiments have shown that these deviable rays are similar in all respects to cathode-rays. Dorn ‡ showed that they were deflected in an electrostatic field, while the Curies § showed that they carried with them a negative charge. Becquerel determined the velocity of these "electrons" by observing the magnetic and electrostatic deviation of the rays. He found that the rays from radium were complex, and had widely different velocities. Some travelled at more than half the speed of light. The ratio of the charge to the mass $\frac{e}{m}$ was found to be about the same as for cathode-rays. These results have recently been confirmed by Kaufmann ‖, who has shown that some of these electrons travel with a speed nearly equal to that of light, while the ratio of $\frac{e}{m}$ is somewhat less than for the comparatively low velocity cathode-rays, and appears to decrease with the velocity of the electron. This points to the conclusion that for these high-speed electrons a portion of the effective mass is electrical in origin ¶.

The authors have found that, in addition to uranium and radium, thorium compounds, and also excited radioactivity due to thorium, give out some rays deviable by a magnetic field.

* Communicated by the Authors. Communicated to the American Physical Society, April 21, 1902.

† See reports on radioactivity by Becquerel and Curie to Congrès International de Physique, 1900, tome iii.

‡ *C. R.* cxxx. p. 1126.

§ *Ibid.* cxxx. p. 647.

‖ See Heaviside, 'Electrician,' April 4, 1902.

¶ *Gött. Nach.* ii. 1901.