by radium B, thorium B, and actinium B. The single penetrating type of radiation for which $\mu = 0.140$ would then be due to uranium X_2. Under these conditions the γ rays emitted by uranium X_1 and uranium X_2 would appear to be quite analogous to those emitted by the other members of the radioactive series.

Attention has already been drawn to the fact * that those radiations which correspond to the "L" series of Barkla seem to persist throughout the whole series of radioactive substances, with the exception of the D products. The ratiation for which $\mu = 24$ and which appears to be emitted by uranium X_1 evidently belongs to the "L" series. This result gives additional evidence that the atomic weight of actinium is about 230 and that actinium is probably a branch product of the uranium series. Fleck † has already pointed out that the branch cannot come from uranium X_2. Now previous work has shown that in the case of radium C and thorium B, branch products occur either immediately before or after the emission of a single penetrating type of γ radiation. If we assume that the actinium branch comes from uranium X_1, then the analogy with the other radioactive bodies would be complete.

Summary.

It has been shown that uranium X, by which are denoted the products uranium X_1 and uranium X_2 in equilibrium, emits three types of γ rays having absorption coefficients in aluminium $\mu = 24$, $\mu = 0.70$, and $\mu = 0.140$ $(\text{cm.})^{-1}$. Evidence has been given that the first two types are probably emitted by uranium X_1 whilst the third penetrating type is due to uranium X_2.

The results obtained support also the view that the atomic weight of actinium is about 230 and that actinium is probably a branch product from uranium X_1.

My best thanks are due to Professor Rutherford for his kind interest and constant help throughout the course of these experiments.

University of Manchester,
December, 1913.

* Rutherford and Richardson, Phil. Mag. xxvi. p. 937 (1913).
† Fleck, Phil. Mag. xxvi. p. 528 (1913).

XXVIII. *On the Structure of the Atom.*

By Professor W. PEDDIE *.

§ 1. *The present Aspect of the Problem.*

THE strongly marked violation of the doctrine of equipartition of energy which is made evident in the phenomena of radiation, together with the violation of the Newtonian dynamics which it seems at first sight to imply, has given rise to the idea of discontinuity of transferences of energy; and a highly approximate empirical representation of the distribution of energy in the spectrum has led to the postulate of a unit of energy as an actual physical entity. But the fact that this unit is a variable, dependent on the frequency, naturally raises doubts as to the physical reality of units separately or superposably located in æther. Though the existence of energy, in association with particular phenomena, in definite multiples of definite units, is an experimental fact, an obviously alternative supposition to the above one is that any emission or absorption of energy in multiples of definite quantities which *actually occurs* is due to structural peculiarities of the emitting or absorbing mechanism. This is a point which must not be overlooked in spite of the great success as a working hypothesis, in other departments of physics besides that of radiation, of the idea of unitary transmission of energy. The most recent success in the department of radiation has been made by Dr. Bohr (Phil. Mag. 1913, July, Sept., Nov.) in the application of the $h\nu$ postulate to the deduction of the formulæ for line spectra.

In the course of the discussion on spectra at the 1912 meeting of the British Association, I stated that the origin of spectrum series could be found rather in a complicated structure of the atom itself, than in complexity of structure of configurations of electrons circulating in or round a comparatively simple atomic structure. It is well known that great difficulties beset the deduction of the laws of series on the latter basis. I pointed out that, on the former basis, such difficulties do not exist; that any spectrum can be accounted for; that, with rotating spherical distributions of electricity, variations of spectra under different physical conditions can be accounted for; and that there are sufficient disposable quantities to provide for observance of the laws regulating emission of electrons under the action of Röntgen rays or ultra-violet light. The full discussion is given here.

* Communicated by the Author.

In Dr. Bohr's investigation the characteristic formulæ for series are deduced in a beautifully direct manner, on the postulate of the existence of energy quanta, and in connexion with Rutherford's modification of the type of atom in which electrons are supposed to circulate in rings around a central nucleus. This leads to the usual impossibility of reconciling certain actions with the laws of ordinary dynamics and electrodynamics. In such cases, without specification of the details of a suitable new dynamics, merely on the ground of transference of energy in quanta, very simple explanations of fundamental phenomena in modern physics are obtained. The value of the new ideas as a working hypothesis cannot be denied. But behind all this procedure there lies the root question whether or not the peculiarities, so readily explained on the new ideas, cannot be explained in terms of the ideas of the older physics as consequences of structural conditions. For example, structural conditions can account for the non-entrance, at ordinary temperatures, of more than five of the freedoms of a molecule of a diatomic gas in interchanges of energy amongst the molecules; and thus a violation of the doctrine of equipartition of energy amongst the freedoms can take place without violation of the laws of ordinary dynamics. Similarly, it may be that structural conditions compel, in certain cases, the emission or absorption of energy in quanta without the existence of definite indivisible units of energy, and that they also introduce a limitation on the effective phases or complexions whose full presence is essential to the deduction of the law of equipartition from the doctrine of chance in combination with the Newtonian laws of motion.

Sir J. J. Thomson has recently (Phil. Mag. Oct. 1913) described an atomic mechanism which would account for many of the peculiarities under discussion. Along with a radial action, uniform in all directions round the atom, he postulates another radial action confined to definite tubes. The expulsion of an electron is effected by the absorption of radiant energy under the condition of resonance. The difficulty, whatever it be, of explaining the necessary magnitude of the absorption with resonance seems to exist, in this case, as strongly as in others to which objection has been raised on that ground. The primary purpose in introducing this type of atom was to obtain a ready explanation of the smallness of the amount of ionization produced, per unit volume of a gas, by means of Röntgen rays. The type described below possesses this property and yet enables us to retain the notion of a continuous wave-front. In the former, the

electrons do not revolve around a nucleus; in the latter, as in Dr. Bohr's scheme, they may so revolve. In the latter, bright line emission may arise from the revolution of electrons in the steady state; in Dr. Bohr's scheme there is no emission, or practically none, in steady states, bright lines being due to vibrations induced—in a manner not specified because it is not within the scope of ordinary dynamics—by the passage of the system from one steady state to another.

Though all such possibilities ought to be considered, for they are directly suggested by actual phenomena, it does not seem to me that we are yet under compulsion to forsake the laws of ordinary dynamics in connexion with atomic properties, or the doctrine of a continuous wave-front in æther, or even, apart from magnetic action, the notion of central symmetry in atomic action.

§ 2. *A centrally symmetrical Atom.*

To meet the condition of stable circulation of electrons we must have regions of attractive force. To meet the conditions of expulsion of electrons, *without necessary absorption of much radiational energy*, we must have regions of repulsive force alternating with the former; and the total work done by the forces of repulsion on an ejected electron must exceed that done on it by the attractive forces by the amount necessary in order to account for the speed of ejection. The law of radial variation of force must be such as to account for any observed spectrum series.

Subject to these conditions many different structural arrangements may be postulated. We are not necessarily bound down to the law of repulsion according to the inverse cube of the distance, or to constancy of angular momentum of the electrons. On other bases, different interpretations of the dynamical significance of Planck's constant h from that given by Nicholson would be furnished. But it is of interest to discuss the spherical counterpart of the tubular atom of Sir J. J. Thomson (Phil. Mag. Oct. 1913).

Let there be relatively broad regions in which the law of radial acceleration is

$$m\ddot{r} = \frac{Ae}{r^3},$$

e and m being the charge and mass of an electron. The volume density of a distribution of electricity giving rise to this is $\rho = +3A/4\pi r^4$ outside an interior radius α which includes within it an amount of negative electricity equal to A/α. If now, over the spherical surface of radius $r_n' > \alpha$, there is uniformly distributed a charge A/r_n of positive electricity, while an equal negative charge is distributed over the shell of radius r_n'', where $r_n' < r_n < r_n''$, there will be no resultant force at r_n; and, in the region between r_n' and r_n'', the force will be towards r_n. The frequency of radial vibration of an electron at r_n is ν_n where

$$2\pi\nu_n = \sqrt{\frac{Ae}{m} \cdot \frac{1}{r_n^2}}.$$

Outside r_n'' the field is A/r^3 as before; and the process of construction may be repeated for a whole series of values of n corresponding to any given spectrum series.

To obtain, for example, Balmer's hydrogen series we must determine the r's by the condition

$$\sqrt{\frac{Ae}{m} \cdot \frac{1}{r_n^2}} = 2\pi C\left(\frac{1}{p_1^2} - \frac{1}{p_n^2}\right),$$

where $C = 3\cdot29(10)^{15}$, $p_1 = 2$, $p_n = n$ which is any integer from 3 onwards.

In the region between r_n and r_n'' an electron can circulate with a frequency ν_n' given by

$$2\pi\nu_n' = \sqrt{\frac{Ae}{m} \cdot \frac{1}{r_n^2}} \sqrt{\frac{\xi}{r_n}},$$

where ξ lies between 0 and $r_n'' - r_n$. If ν_n represents a frequency in the visible spectrum, ν_n' must correspond to a frequency far outside the visible spectrum at the red end if $r_n'' - r_n'$ is very small in comparison with r_n, and extending to infinity as the energy of revolution is radiated away. If, further, $r_n'' - r_n'$ is very small in comparison with $r_{n+1} - r_n$ or $r_n - r_{n-1}$, a very small proportion of the electrons which are projected into the atom from the outside can be retained. The great majority are scattered, some at low, some at high, angles. Only those which strike the atom almost centrally can be retained, and of them, indeed, only a small proportion.

Because of the smallness of $r_n'' - r_n'$, the energy of an electron approaching the atom radially must be practically equal to $\pi\sqrt{Aem}.\nu_n$ if it is to reach the region r_n. Identifying this with $h\nu_n$, as Sir J. J. Thomson does (*loc. cit.*), we get $A = 10^{-17}$. Thus the quantum $h\nu_n$ of energy disappears, taking the form of potential energy, before the radiation of frequency ν_n can be excited.

On the other hand, the absorption of a very small amount, $\frac{1}{2}\frac{Ae}{r_n^4}(r_n'' - r_n)^2$, of energy in the form of radiation of frequency ν_n, will liberate the electron, and cause the reconversion of its potential energy into the kinetic energy with which it escapes from the atom. *Thus we obtain a complete explanation of the readiness with which weak ultra-violet light or Röntgen radiation can liberate electrons possessing the quantum of energy.* The quantum is not absorbed from the radiation, it is absorbed from the energy of the electron when it enters the atom and is stored for future use.

Using Whiddington's result that the kinetic energy of a cathode particle, exciting the hardest Röntgen radiation in an atom of atomic weight ϖ, cannot be less than $\frac{1}{2}10^{16}\varpi^2 m$, we get

$$\tfrac{1}{2}10^{16}\varpi^2 m \not> h\nu.$$

Hence $\nu \not< 6\cdot77\,(10)^{14}$, the actual value being $8\cdot2\,(10)^{14}$. In the case of a helium atom we find $\nu \not< 1\cdot08\,(10)^{16}$.

As in Professor Thomson's investigation, the energy of the pulse radiation is proportional to the square of the frequency in consequence of repulsion effectively inversely proportional to the cube of the distance, which is in accordance with observation. That law of repulsion also leads to a formula for the distribution of energy in the spectrum, which has been shown to possess some correspondence with fact (Sir J. J. Thomson, Phil. Mag. July 1910).

A similar construction can be imagined with any desired law of repulsion. In particular the law of repulsion requisite in order to give Planck's law for pulse radiation may readily be expressed by an infinite series. If $F_1(r)$ be the law of repulsion, while $F_2(r)$ is the law of attraction which is superposed upon it in the thin shells of thickness $r_n'' - r_n'$, we have

$$F_2'(r_n)\,e = F_1'(r_n)\,e + 4\pi^2 C^2\left(\frac{1}{p_1^2} - \frac{1}{p_n^2}\right)^2,$$

where p_1 and p_n are the constants of, say, the lines in Balmer's series, C being an absolute constant.

§ 3. *An Atom characterized by axial Rotation.*

In the preceding case, when the collisions are seldom, the spectrum is practically a line or band spectrum. The continuous radiation in the region of very great wave-length will be feeble in comparison with that associated with the lines. But it is possible to have an atomic structure in which the line emission is produced by the revolution of the electrons in the interior of the atom, and not by their linear vibrations about fixed positions. In the preceding case the trouble is that, as radiation proceeds, the kinetic energy of the electron diminishes, and the electron gradually sinks in, with exhaustion of potential energy, to the region of infinitely slow revolution. In so far as this action is concerned, the spectrum of a glowing gas would be continuous. The trouble was avoided by banishment of the continuous part to a non-experimental range. There is only one way of practically avoiding it (see, however, § 5).

The energy of the radiation must be drawn from an internal store in the atom, the magnitude of which is very large in comparison with the circulatory energy of the electron itself. The positive electricity in the shell within which the electron circulates stably must itself be in rotation, and, if necessary, drag the electron with it. Thus the period of the circular vibration acquired is to some extent independent of the tangential component of the speed of an electron entering the atom from without. If ν_n is the period associated with one of the shells, the condition for steady revolution of the electron is

$$\nu_n^2 = \frac{F_2(r_n) - F_1(r_n)}{4\pi^2 m r_n} e = C^2\left(\frac{1}{p_1^2} - \frac{1}{p_n^2}\right)^2.$$

Hence the attraction within the shell is proportional to the distance from the centre. At the surface $r = r_n'$, a uniform surface distribution of positive electricity must be located, its amount being that requisite to raise the total charge within to the value

$$\frac{4}{3}\pi\rho_n r_n'^3,$$

where

$$\frac{4}{3}\pi\rho_n = 4\pi^2 \frac{m}{e} C^2\left(\frac{1}{p_1^2} - \frac{1}{p_n^2}\right)^2.$$

At the surface $r = r_n''$, a uniform negative distribution must be placed, its amount being that required to make the force become a repulsion $F_1(r_n'')$: and so on. The shell of positive electricity within the range $r_n'' - r_n'$ rotates round a central axis with angular velocity

$$\omega_n = 2\pi C\left(\frac{1}{p_1^2} - \frac{1}{p_n^2}\right),$$

in which the surface distributions may share.

If now we write

$$\frac{1}{2} m\omega_n^2 r_n^2 = qh\nu_n,$$

where h is Planck's constant and q is a numerical multiplier, we find

$$\frac{1}{r_n^2} = \frac{2\pi^2 mC}{qh}\left(\frac{1}{p_1^2} - \frac{1}{p_n^2}\right).$$

In Balmer's series $p_1 = 2$ and the least value of p_n is 3. Thus the minimum value of r from which bright line radiation arises is

$$\frac{1}{\pi}\sqrt{\frac{2qh}{mC}},$$

and the maximum value is $3/\sqrt{5}$ times larger. With the data $h = 6{\cdot}5\,(10)^{-27}$, $m = 8{\cdot}8\,(10)^{-28}$, $C = 3{\cdot}3\,(10)^{15}$, we get as the minimum and maximum value respectively, on the assumption $q = 1$, the numbers $2{\cdot}13\,(10)^{-8}$ cm. and $2{\cdot}86(10)^{-8}$ cm., which are quite in accordance with estimates of molecular magnitudes. The maximum is only twice as great as the value deduced for hydrogen from the observed density in the liquid state. Agreement would result if $q = 1/4$.

It is of interest to consider conditions under which the value $q = 1/2$ would hold. If we write

$$m\omega_n^2 r_n = k_n r_n e$$

$$\tfrac{1}{2} m\omega_n^2 r_n^2 = 2\pi^2 m\nu_n^2 r_n^2 = qh\nu_n,$$

we find

$$k_n r_n e = \frac{q^2 h^2}{\pi^2 m^2} \cdot \frac{1}{r_n^3}.$$

Thus the attraction within each region n, of small breadth $r_n'' - r_n'$, is the attraction due to the law of the inverse cube of the distance from the centre of the atom. If this law is identical with the law of repulsion in the other regions we get

$$A = \frac{q^2 h^2}{\pi^2 me}.$$

If now we also have

$$m\omega_n r_n^2 = a,$$

i. e. if angular momentum is conserved in the regions n, we get

$$a^2 = 2mqh\nu_n r_n^2 = \frac{q^2 h^2}{\pi^2}.$$

Consequently, when an electron enters the atom from without with speed V, and attains radial rest in the region n, its energy is equally divided into potential energy and energy of circulatory motion, the magnitude of each being $qh\nu$. But the total energy is $h\nu_n$. Therefore $q=1/2$, and the independently found condition $q=1/4$ might result from closer approach of centres, during collision or under attraction in the liquid state, than to one full diameter. The same final result is attainable, however, if the intrinsic strength of the repulsion is three times greater than that of the attraction.

From the expression

$$\frac{1}{r_n{}^2}=\frac{2\pi^2 mC}{qh}\left(\frac{1}{p_1{}^2}-\frac{1}{p_n{}^2}\right),$$

together with the estimated value, $3{\cdot}7(10)^{-13}$ cm., of the diameter of an electron, we can calculate the maximum allowable value of p_n in Balmer's series on the assumption that $r_{n-1}-r_n$ is not less than that diameter. If $p_n=27$, r_n would be about 2.10^{-8} cm.; if $p_n=40$, r_n would be about 3.10^{-8} cm., by the limitations $r_{n-1}-r_n=3{\cdot}7\,(10)^{-13}$ and $q=1$. In accordance with the preceding estimate of the minimum and maximum radii, the value of p_n must lie within these extremes. The greatest observed value is 35. The same values follow if $q=1/2$ and $r_{n-1}-r_n$ equal to $\sqrt{2}$ times the diameter of an electron.

But we must make $r_{n-1}-r_n$ a considerable multiple of the diameter of an electron. It is really $r_n{}''-r_n{}'$ which must not be less than that diameter. If we make the multiple 100, q must be made equal to 10^{-4} to give the same value of p_n. With 35 as the maximum value of n, we now find $r_{38}=2{\cdot}62\,(10)^{-10}$ cm.; and the first line ($n=2$) in the series originates at $r_3=3{\cdot}5\,(10)^{-10}$ cm.

§ 4. *The Magnetic Field and the Magneton.*

In the preceding discussion no account has been taken of the magnetic field due to the rotation of the electrification. The tangential component of the field due to the distribution ρ (§ 2) is

$$\mathrm{H}_t=\frac{4}{3}\pi\sin\theta\int_a^r \rho_1\omega_1 r_1\left(\frac{r_1}{r}\right)^3 dr_1-\frac{8}{3}\pi\sin\theta\int_r^\beta \rho_1\omega_1 r_1\,dr_1$$

where β is the external radius, if we regard the negative core as having no rotation. The condition of equilibrium of

a revolving electron is

$$\mathrm{H}_t e\omega r=\frac{\mathrm{A}e}{r^3}+m\omega^2 r,$$

with the limitation $\theta=\pi/2$. This determines ω as a function of r. The previous condition $\nu :: 1/r^2$ could be satisfied if

$$\mathrm{H}_t=\mathrm{B}\omega,\quad \frac{\mathrm{A}e}{r^4}=(\mathrm{B}e-m)\,\omega^2,$$

but, to attain this, other terms must be added to H_t, say terms due to surface distributions of electricity. We shall presume these to be so arranged as to give shells of small thickness within which stable circulation of an electron is possible. In all other regions the resultant force upon an electron is outwards. The resultant electric force acting upon it may always be outwards, the electrodynamic force alone being inwards.

An electron entering the atom with a moment of momentum opposed to that of the shells determining the field is then necessarily ejected. Ejection is then also the fate of any electron whose path is inclined at more than a very small angle to the equatorial plane of the atom. Even when this condition is satisfied, the angular momentum of the electron must not deviate much from that appropriate to one of the shells within which equilibrium is possible.

Let us presume, for the sake of simplicity, that the regions in which the electricity constituting the atom is in rotation round a common axis are very thin shells. Regarding one of these, of radius r_μ, as being effectively a surface distribution of density σ_μ, the radial component of the magnetic force on the shell of radius r_n is

$$\mathrm{H}_r=\frac{8}{3}\pi\cos\theta\left[\sum_1^{n-1}.\,\omega_\mu\sigma_\mu r_\mu\left(\frac{r}{r_n}\right)^3+\sum_{n+1}^{m}.\,\omega_\mu\sigma_\mu r_\mu\right],$$

where m is the total number of shells. The sign of the quantity in brackets determines the stability of the shell if it be free to alter the direction of its axis.

The magnetic moment of a shell is $\frac{8}{9}\pi^2 r^5\omega\sigma$. If this be an absolute constant, or a small integral multiple of an absolute constant, we have therein a physical basis for the magneton. And if different stable arrangements with direct or reverse alignment of the shells be possible, we have a condition under which fundamental changes in spectra may take place. If the internal field be sufficiently powerful, and it must in general be so regarded since the most powerful external fields at our disposal are effects of the same action greatly diminished by distance, these changes will not readily occur, will occur only under special circumstances, and the effect of an external field on the emission of radiation will be slight. The axes of rotation of the shells will always, apart from slight vibrations due to disturbances caused by electrons or other atoms, be co-linear. The trouble arising from want of parallelism of the elementary magnets, which appeared for example in Ritz's model, does not arise in this system, which thus gives a ready explanation of that constancy of the magneton to which Weiss's work has given a fairly broad basis of experimental support.

No more than one stable condition is possible unless the number of shells exceeds three. In the case of four, if we write $r_\mu^3/r_n^3 = 2\,{}_n A_\mu$, $\omega_\mu \sigma_\mu \nu_\mu = x_\mu$, and α_μ for $3/8\pi \cos\theta$ times the equatorial field acting on the μth shell, we have

$$\begin{aligned}
0 . x_1 + x_2 \quad &+ x_3 &+ x_4 \quad - \alpha_1 = 0,\\
{}_2A_1 . x_1 + 0 . x_2 \quad &+ x_3, &+ x_4 \quad - \alpha_2 = 0,\\
{}_3A_1 . x_1 + {}_3A_2 . x_2 &+ 0 . x_3, &+ x_4 \quad - \alpha_3 = 0,\\
{}_4A_1 . x_1 + {}_4A_2 . x_2 &+ {}_4A_3 . x_3, &+ 0 . x_4 - \alpha_4 = 0.
\end{aligned}$$

If we reverse x_μ we must reverse α_μ. Hence no case with one x reverse and the others direct is possible.

Now consider the moments of all the shells to be equal and write $x_2 = px_1$, $x_3 = qx_1$, $x_4 = rx_1$. The conditions are

$$\begin{aligned}
&0 + p &+ q \quad &+ r &- \alpha_1' = 0,\\
&p^{\frac{3}{4}} + 0 &+ q \quad &+ r &- \alpha_2' = 0,\\
&q^{\frac{3}{4}} + \left(\frac{q}{p}\right)^{\frac{3}{4}} &+ 0 \quad &+ r &- \alpha_3' = 0,\\
&r^{\frac{3}{4}} + \left(\frac{r}{p}\right)^{\frac{3}{4}} &+ \left(\frac{r}{q}\right) &+ 0 &- \alpha_4' = 0,
\end{aligned}$$

and

$$1 > p > q > r.$$

With the second and third terms, and therefore α_2' and α_3', negative, the first equation cannot be satisfied; with the second and fourth terms, and therefore α_2' and α_4', negative, the second equation cannot be satisfied. Similarly, if the third and fourth terms are negative, the third equation cannot be satisfied; and the first condition prevents the second, third, and fourth terms from being negative simultaneously. So only one stable arrangement (all terms involving p, q, r positive) is possible with four, as with three, shells provided that the magnetic moments of the four are equal. But a second stable arrangement can be found if the moments are not all equal. Let the ratios of the moments be $1 : k : k' : k''$. If r_1, r_2, r_3, r_4 be the radii of the shells, the conditions are

$$\begin{aligned}
&0 &&+ k\left(\frac{r_1}{r_2}\right)^4 &&+ k'\left(\frac{r_1}{r_3}\right)^4 &&+ k''\left(\frac{r_1}{r_4}\right)^4 - \alpha_1' = 0,\\
&\left(\frac{r_1}{r_2}\right)^3 + &&0 &&+ k'\left(\frac{r_1}{r_3}\right)^4 &&+ k''\left(\frac{r_1}{r_4}\right)^4 - \alpha_2' = 0,\\
&\left(\frac{r_1}{r_3}\right)^3 + &&\left(\frac{r_2}{r_3}\right)^3 + &&0 &&+ k''\left(\frac{r_1}{r_4}\right)^4 - \alpha_3' = 0;\\
&\left(\frac{r_1}{r_4}\right)^3 + &&\left(\frac{r_2}{r_4}\right)^3 + &&\left(\frac{r_3}{r_4}\right)^3 + &&0 \quad - \alpha_4' = 0.
\end{aligned}$$

Taking the third and fourth terms as negative, and therefore α_3' and α_4' negative, we have, for example, with $r_1/r_2 = 0{\cdot}98$, $r_1/r_3 = 0{\cdot}5$, $r_1/r_4 = 0{\cdot}49$, $k' = 1$, $k = k'' = 5$, a stable arrangement *which is non-magnetic*.

We have thus a model of an atom which may be either magnetic or non-magnetic. Although it may be the case that most examples of the non-magnetic condition of substances which can exhibit magnetic quality may arise from counteraction of the effects of individually magnetic atoms, the possibility of counteraction of the effects within the atom itself must be considered.

In the preceding discussion the magnetic action of the electrification within the thin shells has not been entered upon. The distribution of electrification within them may be such as to give rise to satellite lines and the observed peculiarities of the Zeeman effect.

Radioactivity may be caused by slowing down of the angular velocities of the shells producing re-arrangement of the alignment of axes. Under sufficient shock it might be conceivably possible for ejection of a series of shells to take place—an α particle or helium atom, for example, being driven out.

One different type of radiation might arise from vibrations of the axes of the individual shells, another from displacement of their centres.

The proof of the statements made on p. 265 regarding equilibrium and ejection of atoms is as follows. The expression for the radial and tangential components of the magnetic field being respectively written as $N\cos\theta$ and

M sin θ, radial equilibrium ensues if

$$\frac{Ae}{r^3} - Me\omega r \sin^2\theta + m\omega^2 r \sin^2\theta = 0,$$

and tangential equilibrium ensues if

$$Ne = m\omega,$$

or if $\theta = 0$ or $\theta = \pi/2$. If at $\theta = \pi/2$ there is equilibrium, the former condition becomes

$$\frac{Ae}{r^3} + m\omega^2 r = Me\omega r = H_t e\omega r,$$

as already found, and the radial force is one of repulsion at any other value of θ with the same r. Whenever ω slows down sufficiently by radiation, expulsion follows. It is quite possible to arrange equilibrium at a value of θ other than $\pi/2$, but continuous decrease of ω would then occur with continuous increase of θ.

§ 5. *Addendum.*

In order to account specially for the laws observed to regulate the expulsion of electrons by photoelectric action or by Röntgen rays, the requisite law of electrostatic repulsion has been adopted, and no special case of electric attraction has been considered in conjunction with the electromagnetic action. A paper by Professor Conway has appeared in a recent issue (Dec.) of the Philosophical Magazine, in which results of electrostatic attraction have been considered, and a very remarkable mode of preserving constancy of period has resulted, while emission of radiation may proceed by quanta. Such emission does not occur in the cases considered above, but there is no experimental basis necessitating provision for this condition in bright line emission. If such a basis arose electrostatic attraction might have to be postulated in the regions of stable revolution of electrons.

The fundamental difference between Professor Conway's scheme and the above one lies in the origin of stable regions. He seeks it in nodal oscillations of the atom, that is in the qualities which determine atomic vibrations. In the above treatment, it is sought for in the qualities of electricity or æther which determine, it may be, loci of permanent strain, or, in any case, features of permanent atomic structure.

XXIX. *Energy required to Ionize a Molecule by Collision.*

By J. S. Townsend, *Wykeham Professor of Physics, Oxford* *.

In the theory of ionization of gases by collision, it has been shown that the quantity α, representing the number of molecules ionized by one electron in moving through a centimetre of the gas at a pressure of one millimetre, is given by an equation of the form $\alpha = N\epsilon^{-\frac{NE}{X}}$ for the larger values of the force X. The agreement between the formula and the experimental results was much closer than might have been expected, considering the assumptions which were made in finding the formula. Thus, for example, at the lower values of X, it has been found that the velocity of agitation of the electrons exceeds the velocity acquired under the force.

If the quantity α represents the number of collisions with molecules in which the velocity of the ion exceeds a certain value V′, the velocity V′ must be obtained on a different principle from that previously adopted, when the force X is small†. This may be illustrated by taking the values of α recently obtained by Wheatley ‡ for air corresponding to small values of the ratio X/p. The velocity of agitation u, and the velocity in the direction of the electric force W, for these values of X/p, have also been determined§.

For air at 1 millimetre pressure the values of α, u, W, and k are given in the following table, k being the factor by which the kinetic energy of the electrons exceeds that of the surrounding molecules.

X.	α.	$u \times 10^{-7}$.	$W \times 10^{-6}$.	k.
40	·019	9·1	15·0	81
50	·055	10·1	17·3	100
70	·212	11·3	22·0	125
90	·495	12·2	25·5	150

The velocity of agitation of the molecules may be neglected in comparison with that of the electrons, and the mean free path l of an electron moving in air at 1 millimetre pressure may be taken as ·032 centimetre.

* Communicated by the Author.

† See 'Theory of Ionization of Gases by Collision,' p. 28.

‡ F. W. Wheatley, Phil. Mag. Dec. 1913.

§ J. S. Townsend & H. T. Tizard, Proc. Roy. Soc. lxxxviii. p. 336 (1913).