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Superluminal (but causal) propagation of wave packets in transparent media with inverted atomic populations

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The propagation of limited-bandwidth signals, such as Gaussian wave packets, tuned to a transparent spectral region far below the resonance of an inverted two-level atomic medium, can be superluminal, i.e., with phase, group, and energy velocities all exceeding the vacuum speed of light c. Causality is not violated, however. Little distortion and gain can accompany this propagation.

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Sommerfeld and Brillouin [1] treated a long time ago the problem of propagation of light in dispersive media. They pointed out that the group velocity in regions of anomalous dispersion near an absorption line can exceed the vacuum speed of light, and at certain frequencies can become infinite, or even negative. Garrett and McCumber [2] showed that Gaussian wave packets propagating in these regions can in fact propagate at these abnormal group velocities. Within certain realistic approximations, they showed that an incident Gaussian wave packet will be reshaped by the absorption process (in which later parts of the wave packet are attenuated to a greater extent than the earlier parts), in just such a way as to produce a smaller, but undistorted, Gaussian wave packet at the exit face of the medium. Thus the transmitted wave packet has essentially the same shape and width as that of the incident wave packet. The peak of the wave packet appears to have moved at an abnormal group velocity inside the medium. Chu and Wong [3] verified that this unusual behavior actually occurred for weak picosecond laser pulses propagating near the center of the bound A-exciton line of a GaP:N sample.

Recently, we have verified experimentally that a similar effect occurs in the tunneling of wave packets through a barrier [4]. We showed that single-photon wave packets tunnel superluminally through a one-dimensional photonic band-gap material in the forbidden midgap region. Again, the transmitted wave packets were Gaussian in shape, and although much smaller in amplitude, they had essentially the same shape and width as the incident wave packets. We observed that the peaks of these tunneling wave packets appeared on the far side of the tunnel barrier *earlier* than the peaks of control wave packets, which had propagated through air instead of the barrier. Our results are consistent with the theoretical predictions of MacColl [5], Eisenbud [6], and Wigner [7] for the tunneling time based on the stationary-phase method, and also with the prediction of Büttiker [8] for the tunneling time based on the Larmor-precession method. As in Garrett and McCumber's theory, these theories also predicted the superluminal appearance of the peaks of wave packets on the far side of the barrier or medium. Again, without any violation of causality, one can again understand this as a pulse-reshaping phenomenon.

Here I consider the possibility of superluminal propagation of wave packets in a transparent spectral region far below the resonance of an *inverted* two-level atomic medium. Although only the special case of the region near dc will be considered in detail here for clarity's sake, it should be emphasized that the superluminal phenomena discussed here can occur quite generally in other transparent spectral regions of media that possess gain. Quotation marks will henceforth be used on "superluminal" to emphasize that there is no genuine violation of causality, as Sommerfeld and Brillouin's *front velocity* in this medium never exceeds the speed of light in the vacuum. However, in contrast to Sommerfeld and Brillouin, who considered discontinuous (step-modulated) signals, I consider here the propagation of smooth, limited-bandwidth wave packets; e.g., Gaussian ones. The resulting "superluminal" propagation of these wave packets can again be thought of as a pulse-reshaping phenomenon without any violation of causality, since there is no information contained in the peak of a smooth wave packet that is not already contained in its forward tail. However, in contrast to the previous cases of anomalous dispersion and of tunneling, here there is no attenuation of the wave packet after its propagation through the inverted medium, since this medium is essentially transparent when it is excited sufficiently far off resonance. One goal of this paper is to help correct the common misconception that the group velocity, when it is "superluminal," is somehow unphysical or unuseful [9–11].

Consider a gas of *inverted* two-level atoms in a cell of length L. Let us treat the electromagnetic field classically. Let a limited-bandwidth wave packet, whose carrier frequency ω_c is far below the resonance frequency ω_0 of the two-level atoms, be incident on this medium. Let the amplitude of this wave packet be sufficiently small so that only the *linear* response of the medium need be considered. Then it is helpful to use a generalized Lorentz model, in which the oscillator strength f is replaced by its negative $-f$ to characterize the inverted two-level system, since this model is a good approximation to the density-matrix equations of motion for the weakly per-

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turbed two-level system [12]. However, it is important to note that these results do not depend in any crucial way on the Lorentz model; rather, the crucial assumptions here are (1) that the system responds *linearly* to the classical electromagnetic field, and (2) that this response is *causal,* so that the Kramers-Kronig relations are valid.

From the generalized Lorentz model, the refractive index of this inverted medium is

$$n(\omega)=\left[1-\frac{\omega_p^2}{\omega_0^2-\omega^2-i\gamma\omega}\right]^{1/2}, \quad (1)$$

where γ is a (small) phenomenological linewidth and ω_p is the effective plasma frequency [13],

$$\omega_p=(4\pi Ne^2|f|/m)^{1/2}, \quad (2)$$

where N is the atomic number density, e is the electron charge, and m is the electron mass. In typical situations, the inequalities $\gamma \ll \omega_p \ll \omega_0$ are obeyed. Note that the minus sign in front of the second term under the square root in Eq. (1) arises from population inversion: It differs from its usual positive sign for a two-level atomic system in its ground state. As a result of this sign change, the index of refraction near zero frequency is less than unity:

$$n(0)=(1-\omega_p^2/\omega_0^2)^{1/2}<1 . \quad (3)$$

Figure 1 shows a plot of the real part of the refractive index calculated from Eq. (1), versus the frequency ω of an applied monochromatic plane wave.

The fact that $n(0)<1$ implies that the phase velocity

$$v_p(0)=c/n(0)>c \quad (4)$$

is greater than the vacuum speed of light c. More surprisingly, near zero frequency, the group velocity

$$v_g(0)=\frac{c}{\left[n(\omega)+\omega\frac{dn}{d\omega}\right]_{\omega\to 0}}=\frac{c}{n(0)}=v_p(0)>c \quad (5)$$

is equal to the phase velocity near zero frequency, and is therefore also "superluminal," i.e., it also exceeds the

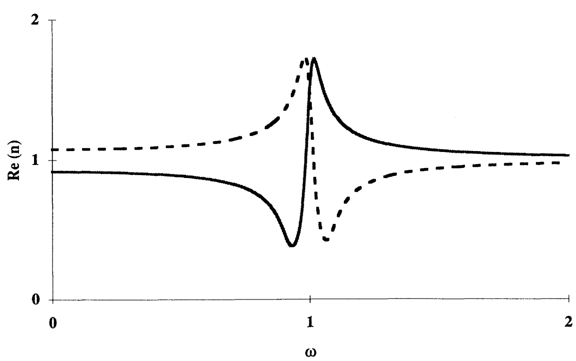

FIG. 1. The real part of the refractive index for an inverted two-level atomic medium (solid line) calculated from Eq. (1) as a function of normalized frequency ω/ω_0, compared with that for an uninverted medium (dashed line).

vacuum speed of light c. Furthermore, in contrast to the *absorptive* media considered by Sommerfeld and Brillouin, since an *inverted* medium can temporarily give up part of its energy to the forward tail of a signal, here the energy velocity is also "superluminal,"

$$v_E(0)=\frac{S}{u}=\frac{c}{\sqrt{\epsilon(0)}}=\frac{c}{n(0)}=v_p(0)>c , \quad (6)$$

where S is the time-averaged Poynting vector, u is the time-averaged energy density, and $\epsilon(0)$ is the dc dielectric constant. That these velocities are equal to each other is a consequence of the essentially *transparent, dispersionless* nature of the inverted atomic medium near zero frequency, just as in the case of a normal (uninverted) medium near dc (note that $dn/d\omega\approx 0$ at $\omega\approx 0$ in Fig. 1).

To understand which approximations are being made, let us start from the general solution for the electric field $E(z,t)$ in a dispersive medium,

$$E(z,t)=\int_{-\infty}^{\infty} A(\omega)\exp\{i[k(\omega)z-\omega t]\}d\omega , \quad (7)$$

where $k(\omega)=n(\omega)\omega/c$, with $n(\omega)$ given by Eq. (1), and where $A(\omega)$ is a limited-bandwidth Fourier amplitude peaked around the carrier frequency ω_c, which specifies the shape of the incoming wave packet at the entrance face of the cell at $z=0$. The limited bandwidth of $A(\omega)$ allows us to perform the Taylor expansion

$$k(\omega)=k(\omega_c)+\left[\frac{dk}{d\omega}\right]_{\omega_c}(\omega-\omega_c)+\frac{1}{2}\left[\frac{d^2k}{d\omega^2}\right]_{\omega_c}(\omega-\omega_c)^2+\cdots , \quad (8)$$

$$\left[\frac{dk}{d\omega}\right]_{\omega_c}=\frac{1}{c}\left[n(\omega)+\omega\frac{dn}{d\omega}\right]_{\omega_c}=\frac{1}{v_g}, \quad (9)$$

$$\left[\frac{d^2k}{d\omega^2}\right]_{\omega_c}=\frac{1}{c}\left[2\frac{dn}{d\omega}+\omega\frac{d^2n}{d\omega^2}\right]_{\omega_c} . \quad (10)$$

At the exit face of the cell at $z=L$,

$$|E(z=L,t)|^2\approx\left|E\left[z=0,t-\frac{L}{v_g}\right]\right|^2 , \quad (11)$$

provided that the dispersion of the wave packet arising from the quadratic term in the Taylor expansion, Eq. (8), is negligible, i.e., provided that

$$\left|\frac{1}{2}\left[\frac{d^2k}{d\omega^2}\right]_{\omega_c}(\Delta\omega_c)^2L\right|\ll 1 , \quad (12)$$

where $\Delta\omega_c$ is the bandwidth of $A(\omega)$. This condition, with Eq. (1) and the inequalities $\omega_c\ll\omega_0$ and $\omega_p\ll\omega_0$, leads to the restriction (same as for the uninverted medium) on the cell length L,

$$L\ll\frac{2}{3}\frac{c\omega_0^4}{\omega_p^2(\Delta\omega_c)^2\omega_c} , \quad (13)$$

for propagation with little distortion. Since the inequalities $\omega_p\ll\omega_0$ and $\Delta\omega_c<\omega_c\ll\omega_0$ hold in realistic situations, the inequality in Eq. (13) is not a severe restriction on L. If we also stipulate that the gain at the carrier frequency ω_c from Eq. (1) be small, there is a further restriction on L, namely,

$$L \ll c\omega_0^4/\omega_p^2\gamma\omega_c^2 \ . \tag{14}$$

If we choose $\gamma \ll \Delta\omega_c^2/\omega_c$, then the little-gain restriction, Eq. (14), is even less severe than the little-distortion restriction, Eq. (13). In addition, for "superluminality" to make sense, it is required that the peak of the "superluminally" propagated wave packet be well resolved (e.g., by Rayleigh's criterion) from that of a wave packet that has propagated at the vacuum speed c, i.e.,

$$\frac{1}{2\Delta\omega_c} \ll L\left[\frac{1}{c}-\frac{1}{v_g}\right] . \tag{15}$$

This condition, with Eq. (1) and the inequalities $\omega_c \ll \omega_0$ and $\omega_p \ll \omega_0$, leads to the restriction

$$\frac{c\omega_0^2}{\Delta\omega_c\omega_p^2} \ll L \ . \tag{16}$$

If we choose $\Delta\omega_c \approx \omega_c$, we see that the inequalities, Eqs. (13) and (16), are consistent with each other, provided that $\omega_c \ll \omega_0$. If we choose $\Delta\omega_c < \omega_c$, the ratio of upper and lower bounds on L is increased still more, so that these inequalities are even less restrictive. Let us choose a cell length L that satisfies these restrictions. Then Eq. (11) will show that *any* limited-bandwidth signal satisfying the above inequalities will be transmitted with little distortion, with near-unity transmission, and with $v_g > c$, i.e., "superluminally." Since there is no information carried by the peak (or peaks) of any limited-bandwidth signal (not necessarily Gaussian) that is not already present in its forward tail, there is no violation of causality.

To show this, and to show that the above results hold in a general way, let us begin from the Kramers-Kronig relations, which assume only *linearity* and *causality* in the response of the medium. The complex linear susceptibility $\chi(\omega)$ obeys the pair of relations [14]

$$\mathrm{Re}\chi(\omega)=\frac{2}{\pi}\mathrm{P}\int_0^\infty \frac{\omega'\mathrm{Im}\chi(\omega')}{\omega'^2-\omega^2}d\omega' \ , \tag{17a}$$

$$\mathrm{Im}\chi(\omega)=-\frac{2\omega}{\pi}\mathrm{P}\int_0^\infty \frac{\mathrm{Re}\chi(\omega')}{\omega'^2-\omega^2}d\omega' \ , \tag{17b}$$

where P denotes the principal value. The first of these relations, Eq. (17a), yields the zero-frequency sum rule

$$\epsilon(0)=1+\frac{2c}{\pi}\int_0^\infty \frac{\kappa(\omega')}{\omega'^2}d\omega' \ , \tag{18}$$

where

$$\kappa(\omega)=\frac{4\pi\omega}{c}\mathrm{Im}\chi(\omega) \tag{19}$$

is the absorption or the gain coefficient of the medium, depending on the sign of $\mathrm{Im}\chi(\omega)$. In the case of an inverted two-level atomic medium,

$$\mathrm{Im}\chi(\omega)<0, \text{ or } \kappa(\omega)<0 \ . \tag{20}$$

From the zero-frequency sum rule, Eq. (18), it follows that the dielectric constant near zero frequency, $\epsilon(0)<1$, is less than unity, and hence that the index of refraction near zero frequency, $n(0)<1$, is also less than unity. Therefore the phase, group, and energy velocities near zero frequency all exceed the vacuum speed of light, as before. However, since one has now proved this beginning from the Kramers-Kronig relations, it is clear that this result does not depend on the validity of the Lorentz model, nor on that of the two-level model. Also, it is clear that causality is not violated.

The Kramers-Kronig argument demonstrates that *in principle* that phase, group, and energy velocities can all be "superluminal" far below the resonance of an inverted two-level atomic medium, without violating causality [15]. However, the infinite-frequency sum rule implies that any real medium will have spectral regions with absorption as well as gain. Nevertheless, it should be possible in practice to observe "superluminal" effects close to, but outside of, a strong gain line. For example, a sufficiently strong, low-frequency gain line can give the dominant contribution to the zero-frequency sum rule, in spite of strong absorption lines at higher frequencies. As a second example, for the case of two nearby gain lines of comparable strength, Steinberg and Chiao [16] have shown that between these lines there exists a point with zero group-velocity dispersion, so that dispersionless "superluminal" propagation is possible there. As a third example, within a bandwidth of approximately half the effective plasma frequency on both sides of a strong gain line, Bolda, Chiao, and Garrison [17] have recently shown theoretically that *negative* group velocities should be observable. We propose a specific system in which an experiment can be done. A computer simulation shows that an off-resonance wave packet that enters a gain medium at the entrance face at $z=0$ generates a transmitted wave packet at the exit face at $z=L$, whose peak leaves the exit face of the cell *before* the peak of the incident wave packet arrives at the entrance face, similar to the negative group-velocity phenomenon observed by Chu and Wong [3] in the context of absorptive anomalous dispersion. There is a backward-propagating, unattenuated wave packet inside the medium generated simultaneously with the transmitted wave packet at the exit face, which propagates backward through the cell and *annihilates* with the arriving incident wave packet at the entrance face. (All *three* wave packets, i.e., incident, backward, and transmitted, have the same shape, but the carrier propagates *forward* inside the backward-propagating wave packet.) There is no violation of the conservation of energy: Since the medium is inverted, it can "loan" energy towards the generation of the transmitted and backward wave packets at the exit face, in just such a way that this energy is "paid back later" by the annihilation of the incident and backward wave packets at the entrance face. After all these interactions are over, the medium is left in its original inverted state, and the final electromagnetic field configuration asymptotically consists of a single transmitted wave packet with the same energy as the incident wave packet propagating to $z=+\infty$.

These predicted "superluminal" effects are different from the subluminal pulse reshaping phenomena that arise from the propagation of laser pulses *inside* the gain band [2,18]. Here we are *outside* of the gain band, in a spectral region where the medium is essentially transparent. They are also different from the off-resonance group-velocity-changing effects in transparent spectral regions next to an *absorption* line [19,20]. Here instead of the absorbing, subluminal case, we have considered the inverted, "superluminal" case. (Of course, the subluminal case poses no potential conflict with relativity, whereas the same is not true of the "superluminal" case, which makes it the more important one to understand.) Also, these effects are different from the previously observed abnormal group velocities of wave packets propagating inside an absorption band discussed above [3]. Here the transmitted wave packets are unattenuated; furthermore, the *energy velocity* can now also be "superluminal." These effects are also different from the recently observed phenomenon of "superluminal" tunneling of wave packets [4], for the same reasons. We believe that they may be similar to a recently observed "superluminal" nonlinear pulse-reshaping phenomenon [21], although there is some ambiguity as to the question of causality in this last case, and the authors attribute their effect to a purported violation of the Kramers-Kronig relations.

An open theoretical issue is how to provide a microscopic description of these effects by means of a fully second-quantized treatment, which could predict the behavior of off-resonance *single-photon* wave packets interacting with the inverted two-level atomic medium. The role of spontaneous emission in these effects can then be clarified [22]. We are also planning to do experiments to demonstrate these striking phenomena.

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[15] Recently, Bolda, Chiao, and Garrison have proved two general theorems based on the Kramers-Kronig relations stating that for *any* dispersive medium, abnormal (i.e., superluminal, infinite, or negative) group velocities must exist at some frequency, and that at the frequency at which the attentuation (or gain) is maximum, the group velocity must be abnormal (or normal).

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