of absorbers is not significantly aligned, so with fifteen different m_J levels, one may only observe a modest broadening in the intermediate-field region.

[12]See for example C. H. Townes and A. L. Schawlow, *Microwave Spectroscopy* (McGraw-Hill, New York, 1955). An expression for rotational magnetic energy is given on page 292.

[13]Anderson and Ramsey, Ref. 11.

[14]H. R. Schlossberg and A. Javan, Phys. Rev. 150, 267 (1966). The possibility of observing an effect of this type with a comment as to its intensity is again mentioned in Ref. 4. It is conjectured there that such effects may have been observed in the I_2 spectrum reported in G. R. Hanes and K. M. Baird, Metrologia 5, 32 (1969). Although the structure in I_2 turned out to have a different origin, the results reported here are a clear confirmation of the effect.

[15]Any detector bias can be estimated by the dissimilarity of signal intensity of the two side peaks of Fig. 2. The Rochon prism may steer one orientation of linear polarization more effectively into the signal detector; a departure from this orientation causes a decrease in detected intensity. It is evident that the bias is small in Fig. 2.

[16]The Doppler-induced resolution of the two cavity waves into two distinct frequencies changes the details of the time averages appropriate to a calculation of the saturation signal. In the low-saturation limit, using the methods of Ref. 2 or Ref. 14, one easily derives a predicted factor-of-two intensity decrease of the saturation signal. This effect operates when the type of resonance being studied requires the absorber to interact with two resolvably different rest-frame frequencies, i.e., where the molecular internal-energy change has two distinct values.

[17]In Ref. 4; and G. Herzberg, *Molecular Spectra and Molecular Structure: II. Infrared and Raman Spectra of Polyatomic Molecules* (Van Nostrand, Princeton, N. J., 1945), p. 455.

[18]See Ref. 12, Sec. 11-6, p. 290 ff.

Pulse Propagation in a High-Gain Medium

Lee Casperson and Amnon Yariv
California Institute of Technology, Pasadena, California 91109
(Received 23 November 1970)

It is found that ultrashort pulses in a high-gain 3.51-μm xenon laser propagate through the amplifying medium at a velocity less than the vacuum speed of light by as much as a factor of 2.5. The pulse velocity is a function of the gain and agrees with the group velocity.

It has been argued theoretically[1,2] that the velocity v of pulse propagation in amplifying or absorbing media is equal to the group velocity $d\omega/dk$. Resonance dispersion may cause the group velocity to be greater (in an absorbing medium) or smaller (in a gain medium) than the phase velocity. These dispersion effects have been observed experimentally[3-5] using the weak 6328-Å transition in neon, but the changes in group velocity were less than 1 part in 1000.

In the following we report on the pulse velocity in a xenon discharge near its amplifying 3.51-μm transition. In this case the combination of high optical gain (>40 dB/m in our experiment) and narrow linewidth result in extremely large dispersion. The observed pulse velocity is less than $\frac{1}{2}c$. Furthermore, using an analytic expression for the gain dependence of the index of refraction of the Doppler-broadened transition, we show that the pulse propagation velocity agrees with the group velocity.

The classical group velocity is

$$v_g = \frac{c}{n + \nu dn/d\nu}. \tag{1}$$

The frequency-dependent index of refraction of an inhomogeneous Doppler-broadened medium has been given by Close.[16] If saturation is unimportant and if the homogeneous linewidth (~4 MHz) is negligible compared to the Doppler width $\Delta\nu_D$, this expression may be written as

$$n(\nu) = 1 + cgF(x)/2\pi^{3/2}\nu, \tag{2}$$

where $F(x)$ is Dawson's integral,

$$F(x) = e^{-x^2}\int_0^x e^{t^2}dt. \tag{3}$$

The frequency is measured by $x = [2(\nu-\nu_0)/\Delta\nu_D] \times (\ln 2)^{1/2}$, and g is the small-signal incremental intensity-gain constant at line center.

Equations (1) and (2) can in principle be combined to obtain the frequency-dependent group velocity. We are most interested in the behavior near line center where $F(x) \approx x$. Then Eq. (2) may be written

$$n(\nu) = 1 + cg(\nu-\nu_0)(\ln 2)^{1/2}/\pi^{3/2}\nu\Delta\nu_D. \tag{4}$$

From Eq. (1) the group velocity is then given by

$$v_g = c(1+\beta)^{-1}, \tag{5}$$

where β is the dispersion parameter

$$\beta = cg(\ln 2)^{1/2}/\pi^{3/2}\Delta\nu_D. \qquad (6)$$

Since β can be much greater than unity in a xenon amplifier,[7] a significant slowing of the pulses should be possible. If the laser were homogeneously broadened, β would be replaced by $\beta' = cg/2\pi\Delta\nu_n$ where $\Delta\nu_n$ is the homogeneous linewidth.

The apparatus used in our experiment consisted of an optical resonator of length $L = 5.5$ m containing an amplifying xenon discharge section of length $l = 1.1$ m. The dc discharge tube was 5.5 mm in diameter and the pressure was maintained at about 5 μm by means of a liquid-nitrogen trap.[8] The mirrors were highly reflecting and the output was obtained from a quartz beam splitter. A xenon laser of this length has a strong tendency to mode-lock spontaneously as has been reported by Kim and Furumoto.[9] We have observed mode-locked pulses with lengths in the range of 5 to 50 nsec depending on the effective width of the gain spectrum. The pulse-repetition rate has been varied from 5 to 50 MHz by varying the cavity length.

A measurement of the pulsation period as a function of gain for a fixed cavity length provides a direct indication of the pulse velocity in the xenon amplifier. The velocity v of the pulses in the amplifying medium is related to the experimentally measured pulsation period T by the expression

$$v/c = [1 + (c/2l)(T - T_0)]^{-1}, \qquad (7)$$

where $T_0 = 2L/c = 37$ nsec would be the pulsation period if the dispersive medium were not present in the cavity. The pulse-retardation effects are so strong in xenon that T_0 can be determined to sufficient accuracy from a simple measurement of the cavity length.

Equation (6) is only valid as long as saturation is negligible. Thus, for a comparison of experiment and theory it was necessary to adjust the losses so that the laser operated very near threshold. Some experimental results are collected in Fig. 1 using Eq. (7) and the measured values of the pulsation period. The gain calibration was obtained by introducing known losses into a laser cavity using the same xenon amplifier and reducing the discharge current until threshold was reached. For fixed losses the retardation was found to decrease with increasing discharge current because of the resultant saturation.

The theoretical curve in Fig. 1 is a plot of Eq.

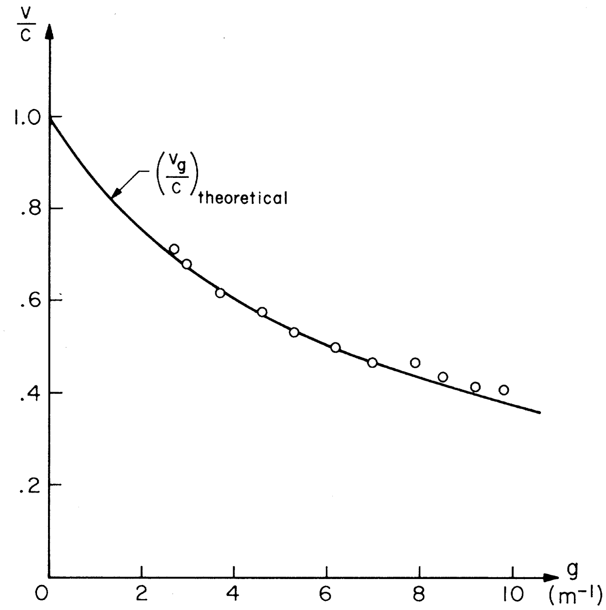

FIG. 1. Gain dependence of the relative pulse velocity. Experimental data are indicated by circles.

(5) with the Doppler width taken as $\Delta\nu_D = 270$ MHz. This value is about twice as large as that resulting from pure Doppler broadening and, as noted earlier,[7] may be partly due to isotope shifts[10] and the hyperfine structure of the transition. The agreement between the data points and the theoretical plot shows that the pulse propagation velocity is indeed given by the group velocity.

In conclusion, we have observed a reduction by a factor of about 2.5 in the propagation velocity of optical pulses due to the dispersion associated with the 3.51-μm transition in xenon. These results are in agreement with theoretical considerations. The pulse slowing is, in fact, the time-domain manifestation of the strong mode pulling discussed previously,[7] and the saturation effects correspond to mode repulsion. The incorporation of a xenon absorbing section into the optical resonator should make possible the observation of values of v_g/c considerably in excess of unity. Experiments in this area are continuing.

[1]E. O. Schulz-DuBois, Proc. IEEE 57, 1748 (1969).

[2]C. G. B. Garrett and D. E. McCumber, Phys. Rev. A 1, 305 (1970).

[3]J. A. Carruthers and T. Bieber, J. Appl. Phys. 40, 426 (1969).

[4]A. Frova, M. A. Duguay, C. G. B. Garrett, and S. L. McCall, J. Appl. Phys. 40, 3969 (1969).

[5]F. R. Faxvog, C. N. Y. Chow, T. Bieber, and J. A. Carruthers, Appl. Phys. Lett. 17, 192 (1970).
[6]D. H. Close, Phys. Rev. 153, 360 (1967), Eq. (44).
[7]L. W. Casperson and A. Yariv, Appl. Phys. Lett. 17, 259 (1970).
[8]D. Armstrong, IEEE J. Quantum Electron. 4, 968 (1968).
[9]H. H. Kim and H. W. Furumoto, Bull. Amer. Phys. Soc. 15, 505 (1970).
[10]R. Vetter, C. R. Acad. Sci., Ser. B 265, 1415 (1967).

Force-Free Configuration of a High-Intensity Electron Beam*

S. Yoshikawa
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 24 December 1970)

The equilibrium configuration of a neutralized electron beam is calculated, including the self-field (parallel to the beam) generated by the beam itself. The resultant equilibrium configuration allows an arbitrary current to be carried by the beam, the motion of electrons being unimpeded by the self-magnetic field. Electrons in this configuration travel almost parallel to the magnetic field; thus, the synchrotron radiation can be minimized. It is suggested that this type of configuration opens up the possibility of low-loss electron coils and, in particular, is useful for fusion reactors.

The limitation for relativistic electron-beam currents (space-charge neutralized) was calculated previously from the condition that the Larmor radius of electrons in the self-field should not exceed the radius of the beam, a. The condition, in terms of magnetohydrodynamics (MHD), is that the equation

$$\vec{J}\times\vec{B}=\nabla W_\perp \tag{1}$$

must be satisfied. Here, $\vec{J}=(0,0,J_z)$ is the beam current density, and $\vec{B}=(0,B_\theta,0)$ is the self-field produced by J; $W_\perp$ is the perpendicular energy density of the electrons, which is variable in order to satisfy Eq. (1) but obviously limited from above by the total energy. From this one gets

$$I^2\sim 4\pi^2a^2W_\perp/\mu_0\leq 4\pi^2a^2W/\mu_0, \tag{2}$$

where I is the beam current (we assume that there is no return current present). Therefore,

$$I\leq 4\pi^2a^2W/I\mu_0\approx 10^4\gamma\beta\ \mathrm{A}\equiv I_A; \tag{3}$$

here the substitution $I\approx\pi a^2nve$ was made (mks), $\beta=v/c$, and $\gamma=(1-\beta^2)^{-1/2}$. A more precise calculation yields $I_A\approx 1.7\times10^4\gamma\beta$ A.[1,2]

By looking at Eq. (1) we immediately notice that there is no reason why $\vec{J}$ is restricted to J_z. Indeed, J_θ can flow in the system to create a component B_z. It is known that with an external field imposed,[2] Eq. (1) can have an I that is beyond the limit imposed by the relation (3). Here we ask, is it possible to arrive at an equilibrium condition for which Eq. (1) can be satisfied, even in the absence of an external magnetic field, for arbitrary current strength I without any abnormal configuration such as that of a hollowed-out beam[2] [the hollowed-out beam still satisfying Eq. (1) with $J_\theta=0$]? The answer is affirmative. Let us, for a moment, ignore the right-hand side of Eq. (1). Then we get the well-known force-free field system and the solution exists in the infinite-beam system. (The question for the finite-beam system is different. Since our interest is the self-consistent field solution, with possible application to a relativistic electron ring, the infinite system is treated here.) We treat the case where background infinite-mass ions are present to neutralize the space change.

The inclusion of the right-hand side, because of the centrifugal force acting on electrons, is now treated. We assume that all the electrons have $v_r=0$. Then each electron must satisfy the relation (m being the relativistic mass of electron)

$$\vec{e}_r m v_\theta^2/r=e(\vec{v}\times\vec{B}), \tag{4}$$

where $\vec{e}_r$ is the unit vector in the radial direction. We assume that there is no velocity dispersion; that is,

$$\vec{J}=-ne\vec{v}. \tag{5}$$

Thus, instead of Eq. (1) we get

$$(\vec{J}\times\vec{B})_r=-m J_\theta^2/ne^2r. \tag{6}$$

Or, using $\mu_0\vec{J}=\nabla\times\vec{B}$ and noting that $\partial\vec{B}/\partial\theta=\partial\vec{B}/\partial z=0$, we arrive at

$$\frac{1}{2}\frac{dB_z^{\,2}}{dr}+\frac{1}{r}\frac{d(rB_\theta)}{dr}B_\theta=\frac{m}{\mu_0nre^2}\left(\frac{dB_z}{dr}\right)^2. \tag{7}$$