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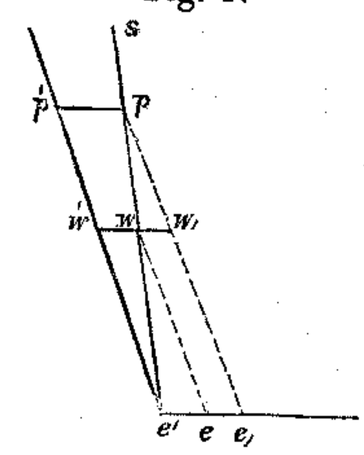
XLIX. *A Theoretical Explanation of the Aberration of Light. By the* Rev. J. Challis, *M.A., Plumian Professor of Astronomy in the University of Cambridge**.

At the meeting of the British Association held at Cambridge last June, I brought forward a theory of the aberration of light, the same in principle as that which forms the subject of this communication, but requiring some elucidation with respect to its applicability on the undulatory hypothesis of light. My present object is to show how it applies on that hypothesis, and for this purpose I proceed to state, first, the general principle of the theory.

Fig. 1.



Let s and w (fig. 1) be simultaneous positions of two visible objects, one of which, s, is fixed in space while the other is carried by the earth's motion with the eye of the spectator. For instance, s may be a star, and w the wire of a telescope. Let the straight line joining the positions s and w be produced to meet the straight line $e_{,}e\,e'$, in which the spectator's eye is moving in the point e'. Take $e'e$ to $e'w$ in the ratio of the earth's velocity to the velocity of light; and let e and w be simultaneous positions of the eye and the object w. Now the light which started from w in the direction we' at the instant the eye was at e, arrives, in company with light from s, at e' when the eye comes to the same point; and the eye receives the impression that w and s are in the *same* direction, because it receives light from each object proceeding in the common

* Communicated by the Author.

direction $s w e'$. But at this instant the object w is at w' ($w w'$ being taken equal and parallel to $e e'$), and the directions of the objects from e' are really separated by the angle $s e' w'$.

Let us now consider two objects which both partake of the earth's motion, and let p, $w_{,}$ and $e_{,}$ be simultaneous positions of the objects and the spectator's eye in the same straight line. Draw $p p'$ and $w_{,} w'$ parallel and equal to $e_{,} e'$, and join $e' w' p'$. Then, since $w_{,} w$ is to $w p$ in the ratio of the earth's velocity to the velocity of light, it follows that the object at $w_{,}$ and the light which started from p in the direction $p w$ when the eye was at $e_{,}$, arrive at w at the same instant. Hence the eye at e' receives at the same time light from p and w coming in the common direction $p w e'$, and consequently sees the two objects in the same direction when they are really at p' and w', and therefore really in the same direction from e'.

It thus appears to be a necessary consequence of the earth's motion and the temporaneous and rectilinear transmission of light, without making any hypothesis about the nature of light or the manner in which the eye receives impressions, that the directions of two objects, one of which partakes of the earth's motion and the other is fixed in space, are separated by a certain angle when they appear to be coincident, while two objects, both of which partake of the earth's motion, are really in the same direction when they are seen in the same direction. The angle of separation, it is plain from the figure, is equal to the ratio of the earth's velocity to the velocity of light, multiplied by the sine of the angle which the direction of the earth's motion makes with that in which the light comes. This result is a complete explanation of the phænomenon of aberration, if only the following remark be added, which, as far as I am aware, has not been before made with reference to this subject. The visual direction of a celestial object is necessarily referred to the visual direction of an object which partakes of the earth's motion, and astronomical observation has discovered that these directions are relatively affected by aberration, but does not determine whether the star or the wire of the telescope is seen out of its true place. We are therefore at liberty to suppose, as the foregoing theory requires, that the apparent place of the *wire* is affected by aberration. It is clear that observations of terrestrial objects alone could not detect aberration (its maximum amount being very small), simply for the reason that two objects partaking of the earth's motion are really in the same direction when they appear to be so, though they may not be seen in their true direction. On this account geodetical observations are unaffected by the aberration of light.

It will be seen that in the above explanation it is assumed that light travels from the object to the eye in a straight line. According to the emission theory of light such is the case, and if this view of the nature of light be adopted, nothing further need be said. The same would be true on the undulatory hypothesis, if we might suppose the æther to be absolutely at rest. But it is impossible to conceive that the earth can move through space without communicating some motion to the æther which surrounds and pervades it. It is therefore necessary, if we adopt the undulatory theory, to inquire how far the preceding explanation will be modified by taking account of the motion of the æther. And here I may remark, that the theory I am explaining is widely different from that which Mr. Stokes has proposed in the July number of this Journal. According to Mr. Stokes's views, the phænomenon of aberration is entirely owing to the motion which the earth impresses on the æther, and which at the earth's surface he supposes to be equal to the earth's motion. On the contrary, I have to show that the amount of aberration will be the same *whatever* be the motion of the æther, and if this cannot be shown, the undulatory theory, and not the foregoing explanation, is at fault. I am, however, so fully persuaded of the truth of the undulatory theory, that I have no doubt this proposition admits of proof, and the following I consider to be satisfactory.

Let, as before, e and w (fig. 2) be simultaneous positions of the eye of the spectator and a terrestrial object, as the wire of a telescope, and let luminous waves from a distant fixed object,

Fig. 2.

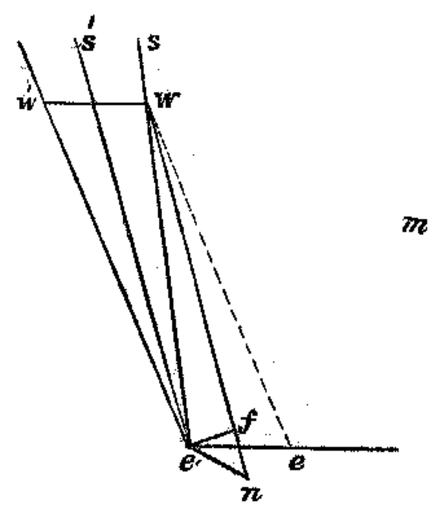
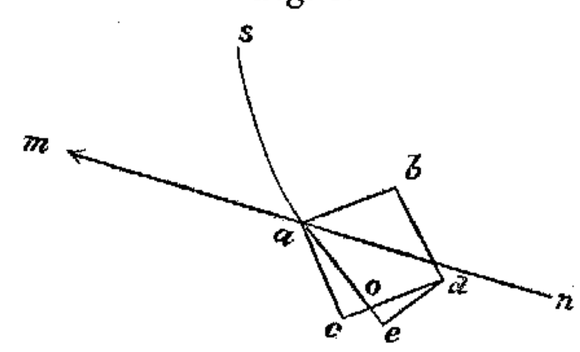


Fig. 3.



as a star, proceed in the direction $s w e'$, accompanied, after reaching w, by waves from w, so that the eye at e' receives the

impression that w and s are in the same direction. Suppose the æther which is situated between e' and w to be moving with a uniform velocity, and let $n e'$ represent in magnitude and direction the space through which it is carried while light travels from w to e'. The line $n e'$ may be of any magnitude less than $e' e$, and in any direction not necessarily in the plane $e w w' e'$. Join $w n$. Now the motion of the luminous waves from w to e' is compounded of two motions represented by $w n$ and $n e'$, the former of which is due to the motion of propagation of the wave through the æther, and the other to the motion of the æther itself. It is well known that the motion of propagation is in the direction of a normal to the front of the wave, and that the front of a given wave continues parallel to a given plane so long as it is propagated through fluid at rest, or through fluid moving with a *uniform velocity in a given direction.* Also the normal to the front of a wave is in the direction of vision. Hence if $e' s'$ be drawn parallel to $n w$, this will be the common direction in which the objects w and s will be seen. Consequently, if the front of the wave retained its parallelism the whole distance from the object to the eye, the *true* direction of the celestial object s from e' would be $e' s'$, when the true direction of the terrestrial object w from e' is $e' w'$. The aberration would consequently be an angle $s' e' w'$, different from the angle $w e' w'$, which is known from observation to be the actual angle of aberration.

It is not, however, true, as we have supposed above, that the front of a wave continues parallel to itself in passing through the æther put in motion by the earth's motion. For evidently the wave is propagated through portions of the æther moving with *different velocities in different directions,* and the effect this circumstance has in altering the direction of the normal to the front of the wave must now be considered. For the following method of calculating this effect, I am indebted to the very ingenious and original mathematical reasoning contained in Mr. Stokes's communication above alluded to. I have only given the reasoning a more geometrical form.

Let $s a$ (fig. 3) be a portion of the path in space of a given point of a wave of constant form, and let a and b be two points of the wave indefinitely near each other in the same phase at the same time. Join $a b$. Since the velocity impressed on the æther by the earth's motion is very small compared to the velocity of light, $a b$ must be nearly perpendicular to $s a$. Let $V_{\prime}$ and $V_{\prime}'$ represent the velocity of the æther at a and b, so far as it is due to the earth's motion, and let that at a take place in the direction $n a m$. Also let V represent the uniform velocity of propagation of the wave through the æther. Then, since the point a with regard to the direction of motion of the æther is *in advance* of the point b, the velocity of the æther is *less* at a than at b. For we are here considering a position in that part of the æther which, with regard to the direction of the earth's motion, is in advance of the earth's centre; and it is plain that if we trace at any time a line of motion in that part, beginning at the earth's surface and proceeding in the direction of the motion of the particles through which it passes, the velocity will be less the further we advance along this line. Let the velocities $V_{\prime}$ and $V_{\prime}'$ continue uniform during the small time δt, and let the small straight lines $a e$, $b d$ be respectively the spaces through which the points a and b of the waves are carried by the composition of these velocities with the uniform velocity V. Join $d e$. Draw $a c$ equal and parallel to $b d$ and join $c d$. It is easy to see that as the motion of propagation is less opposed by the motion of the æther at a than at b, $a e$ is greater than $b d$, and inclined *towards* $b d$. The general effect of the æther's motion is, therefore, to throw the normal to the front of the wave more and more in the direction towards which the æther is moving; and this effect would be similarly found to take place if we considered a position on the other side of the earth's centre.

I proceed now to calculate the amount of deviation of the normal, and to determine the planes in which it takes place. For this purpose let us resolve the velocity $V_{\prime}$ into w along $a c$, u along $a b$, which we may suppose to be in a given direction perpendicular to $a c$, and v along a straight line through a perpendicular to $a b$ and $a c$, and let us first consider the effect of u and w, abstracting from v. The point e will thus be in the same plane as $a b d c$, and if u' be the resolved part of $V_{\prime}'$ in $a b$, we have $c d - d e$, which is very nearly $c o$, equal to $(u' - u)\,\delta t$. Hence if $a c = \delta s$, it follows that the angle $c a o = \frac{(u' - u)\,\delta t}{\delta s}$. But $V = \frac{\delta s}{\delta t}$ very nearly, neglecting the ratio of $V_{\prime}$ to V. Hence the angle $c a o = \frac{u' - u}{V}$, which is the angle of deviation of the normal in the plane $c a b$. The sum of all such angles for the whole course of the wave may be found by taking, since δs is perfectly arbitrary, $a c$ of such a length that u' at c is the same as u' at b; or, which is the same thing, supposing $u' - u = \frac{d u}{d s} d s$. Hence if $u_{\prime}$ be the value of u at the earth's surface, and u_0 at any distant point of the course, the whole deviation in the plane of u and $w = \frac{u_{\prime} - u_0}{V}$. At a

distance much less than that of the moon from the earth, u_0 must be quite insensible. Hence for any celestial body the deviation $= \frac{u_l}{V}$. So the deviation of the normal in the plane of v and w is $\frac{v_l}{V}$. The resulting deviation is therefore $\frac{\sqrt{u_l^2+v_l^2}}{V}$ in a plane containing the direction of the motion of the æther at the earth's surface and the direction of the course of the wave; that is, in fig. 3, the plane $n\,a\,c$, and in fig. 2 the plane $w\,e'\,n$. Now since $n\,e'$ in fig. 2 represents the velocity of the æther at the earth's surface, $e'f$ drawn perpendicular to $w\,n$ represents $\sqrt{u_l^2+v_l^2}$, so that the angle $e'\,w\,f$, which is the same as $s\,e'\,s'$, is equal to $\frac{\sqrt{u_l^2+v_l^2}}{V}$. This then is the *aberration* arising from the different states of motion of the parts of the æther through which the wave is propagated, in consequence of which, when the normal to the front of the wave is in the direction $e'\,s'$, the object is really in the direction $e'\,s$. Thus the actual angle of separation between the directions of the two objects s and w is $s\,e'\,w'$ when they are *seen* in the same direction $e'\,s'$. This angle is the same that we found without considering the motion of the æther, and it therefore follows that the amount and law of aberration are the same whatever motion the earth impresses on the æther.

The foregoing mathematical reasoning does not appear in any respect wanting in generality, and is equally true whether $u\,dx + v\,dy + w\,dz$ be or be not an exact differential. It may, however, be remarked, that as the angle $c\,a\,o$ was found to be equal to $\frac{du}{ds}\delta t$, so we might find the angle $e\,d\,o$ equal to $\frac{dw}{dx}\delta t$, the axis of x being supposed parallel to $a\,b$. But if the form of the wave alters in no respect, the angle $e\,d\,o$ is equal to the angle $c\,a\,o$, and consequently $\frac{du}{ds} = \frac{dw}{dx}$. So $\frac{dv}{ds} = \frac{dw}{dy}$. It would seem therefore that the motion of the æther must be such as to satisfy these equations, at least approximately, the reasoning being only approximate. Now it happens that whenever the motion of an elastic fluid is such that the terms involving the squares of the velocity may be neglected (and the case before us is one of this kind), we have

for determining the motion the approximate equations following:

$$\frac{dP}{dx} = X - \frac{du}{dt}; \quad \frac{dP}{dy} = Y - \frac{dv}{dt}; \quad \frac{dP}{dz} = Z - \frac{dw}{dt}.$$

Hence, supposing no force impressed, we obtain approximately,

$$\frac{d.}{dt}\left(\frac{du}{dy} - \frac{dv}{dx}\right) = 0; \quad \frac{d.}{dt}\left(\frac{du}{dz} - \frac{dw}{dx}\right) = 0;$$

$$\frac{d.}{dt}\left(\frac{dv}{dz} - \frac{dw}{dy}\right) = 0.$$

These equations are true, not because the functions in brackets do not contain t, but because approximately $\frac{du}{dy} = \frac{dv}{dx}$, $\frac{du}{dz} = \frac{dw}{dx}$, and $\frac{dv}{dz} = \frac{dw}{dy}$. Thus the corresponding equations above are accounted for without affecting the generality of the previous reasoning.

Cambridge Observatory, Sept. 29, 1845.

L. *On the Allotropism of Chlorine as connected with the Theory of Substitutions.* By John William Draper, *M.D., Professor of Chemistry in the University of New York**.

[With a Plate.]

THE researches of M. Dumas on chemical types have shown that between chlorine and hydrogen remarkable relations exist, indicating that the electrical characters of elementary atoms are not essential, but rather incidental properties.

The extension of these researches has given much weight to the opinion that the electro-chemical theory may be regarded as failing to account for the replacement of such a body as hydrogen, by chlorine, bromine, oxygen, &c.

I do not know that as yet any direct evidence has been offered that the electrical character of an atom is not an essential quality, but one that changes with circumstances. It appears to be rather a matter of inference than of absolute demonstration.

It is the object of this memoir to furnish such direct evidence, and to show that chlorine, the substance which has given rise to the discussions connected with the theory of substitutions, under the very circumstances contemplated, has its electro-chemical relations changed.

* Communicated by the Author.